Ascending Prices and Package Bidding: A Theoretical and Experimental Analysis*

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We use theory and experiment to explore the performance of multi-round, price-guided, combinatorial auctions. We define efficiency-relevant and core-relevant packages and show that if bidders bid aggressively on these and losing bidders bid to their limits, then the auction leads to efficient or core allocations. We study the theoretically relevant behaviors and hypothesize that subjects will make only few significant bids and that certain simulations with auto-bidders will predict variations in performance across different environments. Testing the combinatorial clock auction (CCA) design, we find experimental support for these two hypotheses. We also compare the CCA to a simultaneous ascending auction.

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I. Introduction

A long tradition in economic thought asserts that prices minimize the amount of communication required in arriving at an efficient allocation of resources. Hayek (1945) emphasized this point as one of the keys to the success of market capitalism, and Hurwicz (1977) formalized it, utilizing the additional assumption that firms' production sets and consumers' preferred sets are convex. Recent theoretical results of Nisan and Segal (2006) make it clear that such additional assumptions are important. In some striking examples, Blumrosen and Nisan (2005) show that price-guided procedures can fail to achieve even a fraction of the maximum possible value. Worst-case theoretical analyses, however, leave open the possibility that auctions guided by individual item prices might perform well in a far wider range of environments than those studied in neoclassical economic theory.

Over the past few years, economic experimenters have explored that possibility. They have designed price-guided auction mechanisms – cousins of the Walrasian *tâtonnement* process – and tested them on selected allocation problems. These new mechanisms differ from the classical Walrasian mechanism in two important ways. First, for any specified prices, the quantities bid at each round represent an actual commitment to buy the demanded bundle at the specified prices. And second, these mechanisms use both current and *past* bids in a single optimization to determine the winning bids, so that formerly losing bids can become winning.[1]

Two recent papers report pioneering experiments with such mechanisms. The RAD mechanism of Kwasnica, Ledyard, Porter, and DeMartini (2005) uses the bids at each round to compute tentative prices for each item that come "as close as possible" to market clearing in a particular metric. After each round, the prices are reported back to the bidders and used to set minimum bids for the next round of bidding. Individual item prices in RAD can increase or decrease from round to round. In contrast, the *combinatorial clock auction (CCA)* mechanism introduced and tested by Porter, Rassenti, Roopnarine, and Smith (2003) determines prices for each item that increase monotonically from round to round.

[1] To illustrate this important feature, suppose that three bidders are bidding for two items, A and B, and that each bidder places only one bid: bidder 1 offers 5 for item A, bidder 2 offers 5 for item B, and bidder 3 offers 11 for the bundle AB, respectively. Then, the highest total price 11 is then obtained from 3's bid alone. If bidder 2 subsequently raises its bid for B to 7, then the highest total price rises to 12, which is obtained by combining 2's new bid with 1's formerly losing bid.

Porter, Rassenti, Roopnarine, and Smith (2003) report surprisingly efficient outcomes from an experiment testing the CCA. In 25 auction trials, efficiencies of 99% are reported in 2 trials and 100% in the remaining 23 trials. Unfortunately, these results cannot be replicated because detailed information about the valuations used in their experiment is unavailable.

Do these experimental findings disprove the significance of the theoretical ones? Can experiments provide a way to assess "average" as opposed to "worst case" performance? Do they enable appropriate comparisons among mechanisms? The answers, we argue, are negative: only a combination of experiments and theory can be sufficient. To illustrate why, consider the experiment reported by Brunner, Goeree, Holt, and Ledyard (2007) and Goeree, Holt, and Ledyard (2007) testing a RAD-inspired design to sell radio spectrum licenses for FCC auction 73. The experiment entailed selling 18 licenses – 12 "national" licenses and 6 "regional" licenses. The number of possible packages is the number of non-empty subsets of a set of 18 items, or 262,143. So, setting aside any possible value interdependencies or externalities, the set of possible value profiles for n bidders is $\Re_+^{262143n}$, which is far too large to explore systematically using experimental trials alone.

In this paper, we study combinatorial auctions using a mix of theory and experiments. Our theory consists of two parts. The first asks the question: under what conditions does a series of bids in a combinatorial auction produce allocations that are efficient or in the core?[2] To answer that, we introduce the concepts of *efficiency-relevant* and *core-relevant* packages and of *sufficiently aggressive* bidding. We show that for a very broad set of auction mechanisms, if bids in the auction for the efficiency-relevant or core-relevant packages are sufficiently aggressive, then the auction outcome will be an efficient or core allocation, respectively. As the relevant packages are only a subset of all possible packages, this conclusion suggests that the most important difference in performance among combinatorial mechanisms may lie in their ability to elicit relevant bids.

The problem of eliciting sufficiently aggressive bids for *relevant* packages is made complicated by the sheer number of profitable packages in some auctions, which we hypothesize makes it likely that some packages never receive any bids. We conjecture that the bids based on

[2] The allocation of an auction is in the core if, in addition to being efficient, it is individually rational and the total price paid by each bidder and group of bidders is "competitive," meaning that it is not less than the opportunity cost of the resources the bidder or group acquires.

price feedback are the only important ones in determining the systematic performance of auction mechanisms like RAD and CCA. This leads us to the second part of our theory – the empirical hypothesis that the performance of simulations with particular automated bidders can predict the performance of auction experiments with human subjects. The automated bidders bid myopically in each round for the currently most profitable package. Our experiment tests this hypothesis using the CCA mechanism and a closely matched version of the simultaneous ascending auction (SAA), which is a non-package auction that is widely used for radio spectrum sales. We use simulations to select parameters for our experiment, including some parameters predicted to lead to efficient auction outcomes ("Easy" cases) and others predicted to lead to inefficient outcomes ("Hard" cases). [3]

Our main experimental findings are mostly consistent with our hypotheses. Subjects consistently bid on only a small number of packages in the CCA auctions: some profitable packages never receive any bids. A large majority of bids are placed on the myopically most profitable packages, but we find that other cues can also influence the packages that subjects choose to bid on. Efficiency is significantly higher in Easy than in Hard cases for the CCA auctions, and the reasons line up well with the first part of our theory: the most profitable packages correspond to the relevant packages in the Easy auctions but not in the Hard auctions. The Easy CCA auctions have significantly higher efficiency than the corresponding SAA auctions, but there are no significant differences in efficiency between the two auction formats for those auctions classified as Hard. Guided by our theory, we look closely at the auctions classified as Hard and identify and characterize a subset of those auctions for which the CCA mechanism generates significantly *lower* efficiency than in the corresponding SAA auctions.

We also divide our test cases into Core-Easy and Core-Hard cases, according to how close the simulated outcomes are to core outcomes. In contrast to the efficiency tests, the distance from the core in simulations is only weakly predictive of the same measure in the experiment. The reasons for this remain to be explored.

For the remainder of this paper: Section II presents our theoretical results, identifying sufficient conditions for obtaining efficient or core allocations in the large class of standard

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[3] We do not report any distribution of selected parameters to subjects, as might be useful if we were testing Bayesian equilibrium theories based on commonly known prior beliefs. Our experiment is not designed to test such theories.

package auctions. Section III describes the experimental procedures. Section IV presents the experimental results and tests our hypotheses. Concluding remarks are offered in Section V.

II. Theory and Hypotheses

To facilitate comparisons of dynamic auctions with direct mechanisms, it is helpful to think of, and sometimes to refer to, the highest bid a bidder makes on a package during the course of a dynamic auction as the bidder's "reported value" for that package. If a bidder never bids for a package, it is as if the reported value is zero.

Our general theory studies two nested classes of auctions. The wider class consists of *total-bid-maximizing* mechanisms, which assign the goods to bidders to maximize the total reported value, that is, the total of the winning bids. An auction is *standard* if, in addition, it fixes bidder payments so that the allocation is in the core with respect to the reported values. Standard auctions include the *menu auctions* of Bernheim and Whinston (1986), dynamic auctions including CCA and RAD, the *ascending proxy auctions* of Ausubel and Milgrom (2002), the *one-shot core-selecting auctions* of Day and Milgrom (2007) (recently adopted for use in Portuguese spectrum sales), and some auction mechanisms recently used in Ireland and the UK.[4] Vickrey auctions, however, are not standard, because their outcomes can fail to be core allocations.

Our theory of standard package auctions emphasizes the role of bids placed on *relevant* packages, and the description of these requires introducing some notation.

Let *N* denote the set of bidders, *G* the set of goods on offer, x_j the package of goods assigned to bidder *j*, $v_j(x_j)$ bidder *j*'s value for its goods, $x = (x_j)_{j \in N}$ the goods assignment, and *X* the set of feasible assignments. The seller is player 0. An *allocation* (x, τ) consists of a goods assignment *x* and a vector $\tau \in \Re^{N+1}$ describing the payments by the bidders and the seller's receipts. The allocation (x, τ) is *feasible* if $x \in X$ and $\tau_0 \leq \sum_{j \in N} \tau_j$. The *total value* of the goods assignment *x* is $\sum_{j \in N} v_j(x_j)$. The triple $\left(N, X, (v_j)_{j \in N}\right)$ defines a *package allocation problem*.

[4] Formally, the simultaneous ascending auction (SAA) without explicit package bids is another total-bid maximizing auction if one specifies that the bid for any package at any round is the sum of the individual bid prices on the lots included in the package. Theorem 2 can then be usefully applied to the SAA as well.

Associated with any package allocation problem is a cooperative game with transferable utility ($N \cup \{0\}$,*w*). The players in this game are the bidders *N* and the seller, denoted as player 0. To define the coalitional value function, it is convenient to introduce notation associating to each coalition some value-maximizing assignment $x(S) \in \arg\max_{x \in X} \sum_{j \in S} v_j(x_j)$. The value of the coalition consisting of the seller and the bidders in set *S* is $w(S) = \sum_{j \in S} v_j(x_j(S))$; any coalition that excludes the seller has value zero.

For any package x_j, let $\beta_j(x_j)$ denote the highest price that *j* bids for that package during the course of the auction, with $\beta_j(x_j) \equiv 0$ if no bid is made on the package. Given such a profile of "reported values" β, we define $x^\beta(S) \in \arg\max_{x \in X} \sum_{j \in S} \beta_j(x_j)$ and $w^\beta(S) = \sum_{j \in S} \beta_j(x_j^\beta(S))$. These definitions are exactly analogous to the definitions of $x(S)$ and $w(S)$ using the "actual" values *v*.

Associated with any allocation is a payoff vector or *imputation* π given by $\pi_0 = \sum_{j \in N} \tau_j$ and, for $j \in N$, $\pi_j = v_j(x_j) - \tau_j$. An imputation corresponds to a feasible allocation if $\pi_0 + \sum_{j \in N} \pi_j \le w(N)$. A feasible allocation (x, τ) is a *core allocation* and the corresponding imputation π is a *core imputation* if π is *individually rational* ($\pi_i \ge 0$ for $i \in N \cup \{0\}$) and satisfies the no-blocking inequalities $\pi_0 + \sum_{j \in S} \pi_j \ge w(S)$ for every set of bidders *S*. We denote by $Core(N, X, v)$ the set of all core allocations for the package allocation problem $(N, X, (v_j)_{j \in N})$.

We can similarly define the *reported* core allocations and imputations by replacing *w* by w^β and using reported profits $\pi_j^\beta = \beta_j(x_j) - \tau_j$. An auction is *total-bid-maximizing* if for every profile of bids β, it selects an assignment $x \in \arg\max_{z \in X} \sum_{j \in N} \beta_j(z_j)$. It is *standard* if, in addition, for every profile of reported values, it selects a reported core allocation $(x, \tau) \in Core(N, X, \beta)$.

We say that a set of bidders S is *core-relevant* if there is some core imputation π such that $\pi_0 + \sum_{i\in S}\pi_i = w(S)$. Then, for each $j \in S$, the package $x_j(S)$ is bidder *j*'s *core-relevant* package. Similarly, the package $x_j(N)$ is *j*'s *efficiency-relevant package*. By a standard property of systems of linear inequalities, the *core imputations* are those satisfying $\pi_0 + \sum_{j\in S}\pi_j \geq w(S)$ for every core-relevant set of bidders *S*. According to the next two theorems, it is sufficiently aggressive bidding (as defined by a certain inequality) on relevant packages that leads an auction to result in core allocations or efficient allocations.

Theorem 1. In a standard package auction, suppose the outcome $(\bar{x},\tau)$ is individually rational and let β denote the final bids in the auction. If for all core-relevant sets of bidders *S* and for all $j \in S$, $v_j(x_j(S)) - \beta_j(x_j(S)) \leq v_j(\bar{x}_j) - \beta_j(\bar{x}_j)$, then $(\bar{x},\tau) \in Core\left(N, X, v\right)$.

Proof. Since individual rationality is given, we need only establish the feasibility and the no-blocking inequalities.

For feasibility, observe that $w(N) \geq \sum_{j\in N} v_j(\bar{x}_j) = \pi_0 + \sum_{j\in N}\pi_j$. The first inequality follows from the definition of *w*(*N*), the second from the definition of the imputations.

For no-blocking, let *S* be a relevant set of bidders. In a standard auction:

$$\begin{aligned}
\pi_0^\beta + \sum_{j\in S}\pi_j^\beta &\geq w^\beta(S) \\
&= \max_{z\in X}\sum_{j\in S}\beta_j(z_j) \\
&\geq \sum_{j\in S}\beta_j(x_j(S)) \\
&\geq \sum_{j\in S}\left(v_j(x_j(S)) - v_j((\bar{x}_j)) + \beta_j(\bar{x}_j)\right) \\
&= w(S) - \sum_{j\in S}\left(v_j(\bar{x}_j) - \tau_j\right) + \sum_{j\in S}\left(\beta_j(\bar{x}_j) - \tau_j\right) \\
&= w(S) - \sum_{j\in S}(\pi_j - \pi_j^\beta)
\end{aligned} \tag{1}$$

The first inequality holds because the auction is standard and selects an allocation $(\bar{x},\tau) \in Core\left(N, X, \beta\right)$. The second line is the definition of w^β and the third follows from maximization. The fourth follows from the hypothesis of the theorem and the last two lines merely rearrange terms and apply definitions. Comparing the first and last terms and recalling that $\pi_0^\beta = \sum_{j\in N} \tau_j = \pi_0$, we have $w(S) \le \pi_0 + \sum_{j\in S} \pi_j$, so the coalitional no-blocking constraints are satisfied. **QED**

Theorem 2. In a total-bid maximizing package auction, let β denote the final bids in the auction and $(\bar{x},\tau)$ the auction outcome. If for all bidders *j*, $v_j(x_j(N)) - \beta_j(x_j(N)) \le v_j(\bar{x}_j) - \beta_j(\bar{x}_j)$, then the goods assignment $\bar{x}$ is efficient: $\sum_{j\in N} v_j(\bar{x}_j) = w(N)$. If the efficient goods assignment is unique, then the condition $v_j(x_j(N)) - \beta_j(x_j(N)) \le v_j(\bar{x}_j) - \beta_j(\bar{x}_j)$ is necessary as well as sufficient for $\bar{x}$ to be efficient.

Proof. We calculate as follows.

$$\begin{aligned}\sum_{j\in N} v_j(\bar{x}_j) &= \sum_{j\in N} \beta_j(\bar{x}_j) + \sum_{j\in N} \left(v_j(\bar{x}_j) - \beta_j(\bar{x}_j)\right) \\ &\ge \sum_{j\in N} \beta_j(x_j(N)) + \sum_{j\in N} \left(v_j(\bar{x}_j) - \beta_j(\bar{x}_j)\right) \\ &\ge \sum_{j\in N} v_j(x_j(N)) = w(N)\end{aligned} \tag{2}$$

The first equality is an identity; the first inequality is justified by the definition of a total-bid-maximizing package auction. The second inequality follows from the hypothesis of the theorem and the final equality follows from the definition of $x_j(N)$. That proves the first assertion.

For the second, suppose that $\bar{x}$ is efficient. Since the efficient goods assignment is assumed to be unique, $\bar{x} = x(N)$, so $v_i(x_i(N)) - \beta_i(x_i(N)) = v_i(\bar{x}_i) - \beta_i(\bar{x}_i)$. **QED**

Informally, the theorems say that if bidders bid sufficiently *aggressively* for their efficiency-relevant or core-relevant packages in some particular auction, then the outcome is efficient or in the core, respectively. The conditions in these theorems are implied by equilibrium conditions in certain complete information auction mechanisms (Bernheim and Whinston (1986), Ausubel and Milgrom (2002), Day and Milgrom (2007)). In the selected Nash equilibria of these earlier papers, bidders bid equally aggressively on all packages, so the conditions of Theorem 1 are satisfied and the equilibrium outcomes are core allocations.

As previously observed, the sheer number of possible packages in a large auction ensures that a bidder can bid on only a subset of its profitable packages. What might guide bidders in such settings to bid on their relevant packages, as the theorem requires? In experiments, various cues could influence bids. A subject might bid for a regional package of spectrum licenses because that package appears to be most profitable at the quoted prices, or because the bidder's role in the experiment is labeled "regional bidder," or because the experiment assigns the bidder low or zero values for licenses outside the named region, or because the bidder enjoys value synergies only among licenses in that region, etc. Yet such cues may be weak and are equally available to bidders in different combinatorial auction mechanisms.

We conjecture that the CCA and similar mechanisms, ones in which the information feedback in each round consists of information about winning bids and proposed prices, can perform consistently well in a class of environments only if bidder strategies using only that information can identify the relevant packages in those environments. To formalize that conjecture, we simulate auction results using automated bidders programmed to bid at each round *only* for the most profitable bundle at the prevailing prices. In some settings, the simulation does lead reliably to efficient or core allocations, but in others, it does not. Our conjecture is then that variations mechanism performance across different environments will be well predicted by the simulation results.

To summarize, our proposed analysis of package bidding thus entails two central hypotheses:

1. *In price-guided package auctions, at each round, bidders bid for many fewer packages than are profitable at the current round prices.* When there are many items for sale, it is typically impossible for bidders to bid on even a small fraction of the full set of packages. But even with fewer items, subjects may be unwilling to spend the effort to bid for many packages. And, it may not serve the bidder's interest to bid for multiple packages, because that may drive up prices for other preferred bundles. In our design, bidders with provisionally winning bids may refrain from bidding at higher prices in the hopes of winning at a low price. Without specifying any single reason, our hypothesis here is that bidders bid for only one or a few packages even in small-scale auctions like those in our experiment.

2. *Simulations in which automated bidders bid only for the currently most profitable package will lead to (near) core or efficient outcomes in the same environments where experimental outcomes lead to approximate core or efficient outcomes.* This hypothesis is based on the idea that much of what subjects do during an auction experiment is irrelevant. The auction outcome is guided mainly by bidders responding myopically in each round to price signals and bidding for the currently most profitable package.

For our experiment, we have identified two features that could subtly affect the experimental findings. First, in most of the cases we examine, the bidders have only one core-relevant package, which makes it much easier for their bids to satisfy the conditions of Theorem 1. Second, in the version of the CCA that we investigate, bidders are informed when they have a provisionally winning bid. That has turned out to be a consequential decision: the provisionally winning bidder status may become a trap if the bidder, perhaps hoping to get a low price, declines to increase its bid while it remains a provisional winner. By the time its status changes and its bid is no longer a provisional winner, prices may have risen too high to make it profitable to bid again. In terms of the theorems, we can describe these situations as failures of bidders to bid sufficiently aggressively for the relevant packages.

In addition to reporting statistics about these two hypotheses, we also report on certain aspects of bidder behavior and we compare the performance of the CCA to a non-package auction alternative, the SAA auction. For bidder behavior, we ask: In our package design, do the provisionally winning bidders stand pat or make new bids? In the non-package design, when there is an exposure problem, how do bidders respond? Do they withdraw, avoiding losses but possibly missing out on potential profits? Or do they continue to bid and risk suffering losses? For evaluating performance, we analyze the auctions in terms of efficiency, revenue, bidder profits, distance to the core, and number of rounds to completion.

Before summarizing our findings, we describe the details of the experimental design.

III. Experimental Design and Procedures[5]

We conducted auctions for either four or six items. Since we use similar value structures in both cases, we give a detailed description only for the six-item case, as illustrated in Figure 1.

There were three bidders in each auction. The north regional bidder had a positive value only for items A, B and C, and earned a positive synergy value in case it acquired two adjacent items, either A and B (henceforth AB) or B and C (henceforth BC). If the bidder acquired ABC, it enjoyed two synergy values. Similarly, there was a south regional bidder with positive value for items D, E, and F, with zero value for the other items, and with identical, positive synergies between items D and E and items E and F. Finally, there was a global bidder with positive value for all six items and identical synergies for all adjacent pairs: AB, BC, DE, EF, AD, BE, and CF.

For our simulations and the experiment, the standalone values for the regional bidders were integers from the interval [5, 75]. There was a single synergy value for each bidder between any pair of adjacent items, which was an integer from [5, 15] in the low synergy regime or from [25, 35] in the high synergy regime. The high or low synergy regime was in place for both regional bidders at the same time and was announced prior to each auction. The standalone values for global bidders were integer values from [5, 45] and synergy values were integers from [25, 35]. An online data file records the actual value profiles used in our experiment.

[5] The full set of instructions, which includes more sample screen shots, is available at http://www.econ.ohio-state.edu/kagel/KLM_instructions.pdf.

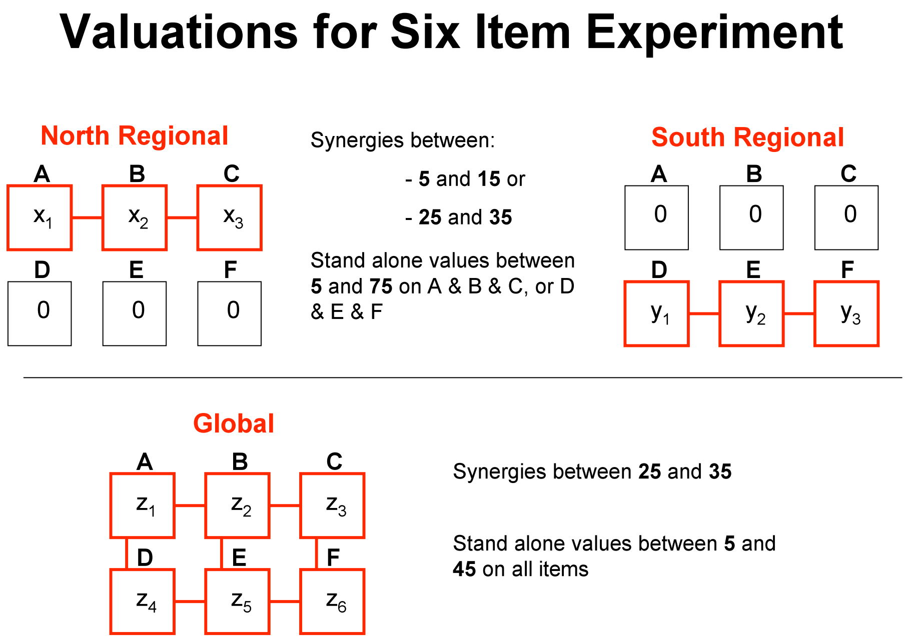

Figure 1

The four-item auctions were the same as the six-item auctions but with standalone items C and F dropped. In both cases bidders knew the auction structure – as they were provided with a copy of Figure 1 as well as a detailed description of the possible synergy relationships and stand alone values. However, in any particular auction they got to see only their own valuations.

After the simulations, we selected valuations for the experiment that we deemed useful for testing hypothesis 2. We chose some cases in which repeated simulations of the CCA were 100% efficient ("Easy" cases) and others in which they were not ("Hard" cases).[6] We then selected certain Easy and Hard cases for the 4-item and 6-item treatments (see Table 5 below for the number of valuations in each category).

During the analysis phase to follow, we also make some additional distinctions. We distinguish between Medium Hard and Very Hard cases depending on the availability of non-price cues that may help bidders to identify their efficiency-relevant packages – a distinction that our theory suggests could influence the efficiency of outcomes. And, in analyzing distances from the Core, we distinguish cases according to how close the simulation outcomes are to core allocations.

[6] Repeated simulations may lead to different results because ties for the high bid are resolved randomly.

CCA Auctions

Our auctions were run using a variant of the CCA rules of Porter et al. (2003). Subjects could bid on one or more packages, such as the package ABC (containing A, B **and** C). Bids for each bidder were XOR bids, meaning that only one of the bids could be a provisionally winning bid in any given round of the auction. In the CCA, when a bid won, the bidder was assigned *all* the items in its winning package and only those items.

In each round, bidders observed the prices for each item and decided about which packages to bid on. Each resulting package bid is a pair consisting of the named package and a single package price equal to the sum of the current round prices of the included items. Subjects could submit as many package bids as they wished. At the end of each round, *tentative* winning bids were determined from among all current and past bids by finding the feasible combination that maximized seller revenue.[7] Prices associated with past bids were based on prices in the round in which the bids were originally placed.

Prices for all items started at 5 ECUs (experimental currency units), and prices were raised according to the following rules. Look at the set of provisionally winning bids in the previous round and the set of new bids in the current round. If an item attracts a new bid from two or more bidders, or if it is named in a provisionally winning bid and a new bid, then its price is raised by 5 ECUs. Otherwise the item price remains the same.[8] Thus, by looking at which items had price increases for the current round, bidders could easily identify items for which others were actively competing.

Following each round, bidders were privately informed about which, if any, of their bids was provisionally winning. [9]

Subjects were encouraged to place bids on multiple potentially profitable packages, particularly early on as "… the opportunity to make profitable bids on individual items or packages with low synergies, which may become provisional winners later in the auction, will

[7] Ties for tentative winning bids, which are to be expected early on in the auction, were broken randomly with priority given to tentative winners in the previous round if prices did not change. Ties become less of a concern in later stages of the auction.
[8] Prices were thus weakly increasing from round to round, unlike RAD (Kwasnica et al. (2005)) or the FCC's Modified Package Bidding.
[9] Tentative winning bids were *not* announced in either the original Porter et al. (2003) experiment or in Brunner et al. (2007).

only be present early in the auction."[10] There were no activity rules restricting the items subjects could bid on.

An auction ended after two consecutive rounds of no new bids or, what amounts to the same thing, no price increases. Two rounds were used to give everyone a chance to determine whether they were satisfied, given current prices, with their provisionally winning allocations.

SAA Auctions

Our SAA screen was designed to look the same as the CCA screen, so that differences in comparative performance could not be attributed to differences in presentation. The rules were also designed to be as similar as possible. Like the CCA, the SAA proceeded in a series of 25-second rounds. And, like the CCA, a subject only had to click "set" next to any set of items to place a bid on those items. However, unlike the CCA, an SAA bidder could only make one such bid and that bid was interpreted and processed as a collection of independent item bids rather than as a package bid.

In each round, for each item with excess demand, the price was increased by 5 ECUs. The auction ended once there was no longer excess demand for any item, and each item was sold at the current price. Thus, a bidder who bid more than his or her standalone value for an individual item in order to capture the synergy payoff was exposed to a possible loss from winning just one item and paying more than its standalone value. Our version of the SAA had a number of rules and features not present in the CCA.

1. *Activity requirement:* Each auction started with bidders eligible to bid on all items – six in this case. In subsequent rounds the total number of items a bidder was eligible to bid on could not exceed the number bid on in the previous round. This *activity rule*, which resembles the rule used in spectrum auctions, was explained to bidders as necessary to have the auction close in a timely manner.
2. *Default bids:* Each round of the auction started with a default bid labeled "currently demanded bid" which was the previous round's bid (or a bid on all items in the first

[10] In a mechanism design experiment, the instructions are an important part of the treatment as bidders are informed of the favorable properties and operation of what will typically be a novel institution.

round of bidding). Any time a new bid was entered that reduced eligibility, the bidder was notified and required to reconfirm the bid.[11]

3. *Minimum bid requirement:* Once there was no longer any excess demand for an item, the current high bidder could not withdraw the bid for that item, with this requirement in effect until someone else topped that bid. This minimum bid requirement held regardless of whether there was a positive profit on the item (or set of items) in question.
4. *Price rollback rule:* Given the indivisibilities inherent in the fixed-price increase rule, near the end of an auction it would not be unusual for two bidders to drop their demand for the same item at the same time, moving from excess demand to zero demand. This could result in unsold items with a potentially large, negative impact on efficiency. A price rollback rule, described in detail in the experimental instructions, was designed to deal with this situation.[12] This rule randomly assigned the item in question to one of the bidders demanding the item in the previous round at the previous round's price.

Computer Interface and Aids for Subjects

Auctions with multiple items and synergies among them are quite complicated for subjects, so the nature of the bidder interface and any analytic tools it includes can affect bidder behavior and hence experimental outcomes. Since we intended the experiment to be representative of high-quality field implementations, we aimed to have as friendly a computer interface as possible, as well as to provide subjects with computational aids they might expect to have from support staff in a field setting. These aids consisted of a table listing *all* possible bids, with corresponding analytic information, so that subjects could bid on items by simply clicking on the "add" or "set" space next to packages they were interested in. To make it easy for bidders to compare alternative packages, the table could be sorted using a number of potentially relevant criteria; e.g., current cost, current profit, etc.[13] A *double-criterion sort* routine was employed so that a bidder interested in comparing a particular group of bids could do so easily. In the case of the six-item auctions, this was adapted so that the sort routines for regional bidders first sorted

[11] An earlier set of SAA auctions showed that without these proactive procedures a number of subjects let their eligibility lapse well before it was profitable to do so. See the online appendix at http://www.econ.ohio-state.edu/kagel/KLM2010AppendixOnline.pdf for a comparison of outcomes in these earlier SAA auctions with the ones reported on here.

[12] The minimum bid requirement would not apply in this case, as there would be no current high bidder for the item in question.

[13] See the online instructions for complete details regarding this and the rest of the bidder aids provided.

based on packages containing only those items with positive values, followed by all remaining packages.[14] The same set of aids was provided for the SAA and CCA auctions.

Experimental Procedures

Subjects were recruited to participate in a series of three sessions taking place within a two-week period, with each session lasting for approximately two hours. Within each series, all of the auctions had the same auction mechanism – SAA or CCA – and the same number of items (four or six). The first meeting was a training session where subjects were introduced to the experimental procedures and computer interface, followed by three dry runs, which were all that could be completed in the initial two-hour period. To insure a high return rate, subjects were offered a $30 participation fee, to be paid only after the completion of all three sessions, and half of session 2's profits from the auctions were similarly withheld until the completion of all three sessions. In addition, subjects were paid a flat $10 at the end of the initial training session in lieu of any earnings from the dry runs. Given the complicated nature of the auctions, subjects were permitted to take the instructions home. Earnings in sessions 2 and 3 were advertised to range between $10 and $60 or more per person with average earnings of $30-$40 per person. Payoffs were denominated in experimental currency units (ECUs), with a minimum conversion rate of 1 ECU = $0.20.[15] Subjects were provided with starting capital balances of 150 ECUs. Any profits earned in an auction were added to these starting capital balances, and losses subtracted from it, with total earnings for a session consisting of a subject's end-of-session balance, less 130 ECUs, but not less than zero.

Subjects' roles as a regional or global bidder were randomly determined prior to each auction, with bidders in each auction group randomly re-matched following each auction. Each experimental session was designed to have five or more auctions (all with the same valuations) running at the same time. In case the number of subjects was not a multiple of three, the extras became observers for that auction, and were guaranteed to be active in the next auction. Subjects' computer screens reported only their own outcome until the end of the auction, when

[14] Automatic check marks for regional bidders were only employed in the six-item auctions. These were initiated on account of the increased number of packages available to bid on. In the four-item auctions, bidders could effectively see all the packages on a single screen so the risk of mistakenly choosing dominated packages was not as severe.

[15] In some sessions, the total subject earnings at the minimum conversion rate was very low, so we adjusted the conversion rate upward after completion of the session.

the full allocation of units to all bidders in their auction was reported along with a final analytics screen that they could play with. The latter was designed to give bidders a chance to see what profitable packages (according to the ending prices) they might have missed bidding on.

Each auction began with a notification to bidders about their valuations. Bidders were then given a couple of minutes to sort packages and to check any items/packages they might be particularly interested in. The six-item auctions started out with each auction round lasting 25 seconds. After round 6 or 7, the round time was reduced to 20 seconds, and it was reduced further to 15 seconds after round 12 or so, to speed things up. Once these shorter round times went into effect, the auctioneer announced "round ending" a second or two prior to the round actually ending.[16]

[Insert Table 1 here]

Table 1 lists the auction sessions conducted, along with the number of subjects and the number of different valuations employed in each session.[17] Subjects were recruited through e-mail lists of students taking economics classes at Ohio State University in academic year 2006-07. For subjects completing all three sessions, average earnings per subject for the six-item auctions were $174, with minimum earnings of $90 and maximum earnings of $331, including the $30 show-up fee and the $10 payment for the first session. Average earnings per subject for the four-item auctions were $125, with minimum earnings of $51 and maximum earnings of $243, including the $30 show-up fee and the $10 payment for the first session.

IV. Experimental Results

By design, our analysis is organized into Easy and Hard groups of parameters. In our CCA simulations, bidders bid on the most profitable packages equally in the Easy and Hard cases, so it will be interesting to test whether experimental subjects behave in the same way. We begin by showing that they do indeed. We then pool data from the two cases to report the

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[16] Four-item auctions, which were conducted first, had fixed round times of 25 seconds. The procedure was changed in anticipation of a larger numbers of rounds in the six-item auctions. Bidders appeared to have no trouble keeping up with this pace of rounds; they were canvassed regarding whether or not they had enough time to bid on all the packages they wanted to both during and after the pilot sessions. Some even complained that the pace was too slow.

[17] There were two sets of pilot experiments, which are not reported for both CCA and SAA auctions. They were used to refine the auction mechanisms so they would run smoothly and quickly, as well as our experimental procedures (e.g., Would it really take most of two hours to go over the software and run a handful of auctions?). The number of valuations employed in each session was determined in advance. The same set of valuations was employed in the same sequence between corresponding CCA and SAA auctions. The number of auctions was not announced in advance.

characteristics of individual bidder behavior that our theory suggests can be consequential for auction outcomes. Finally, we compare the performance of the two auction mechanisms in terms of efficiency, seller revenues, bidder profits, distance from the core, and rounds to completion.

Patterns of Individual Bidding

Subjects' bidding behavior in the CCA auctions exhibits a number of consistent characteristics that our theory identifies as consequential.

First, as hypothesized, bidders bid on only a small number of profitable packages, with the most profitable package attracting the most attention. Further, these patterns do not differ materially between Hard versus Easy auctions. This is potentially important, because if subjects bid on more packages in one of the cases, that would make it more likely that they bid on any particular relevant package in that case.

Table 2 summarizes these data. Columns 2 and 5 of Table 2 report the average number of packages bid on in each round along with the number of profitable packages available to bid on (in parentheses) for global and regional bidders, respectively. The columns following these show where the bids were directed in terms of the percentage of times subjects bid on the most profitable and second-most profitable packages, respectively.[18] Data are excluded for the last two rounds of each auction where by definition there are no new bids as well as rounds in which the bidder is a provisional winner (which will be covered in detail below). So, for example, in rounds 1-5 in the CCA4 auctions, global players bid on 3.9 packages per round on average (out of 13.8 profitable packages available to bid on), of which 76.7% were directed at the most profitable package.

[Insert table 2 here]

As predicted, players bid on only a small number of the profitable packages at each round and omitted bidding on others. They did so even in later rounds when there were relatively few profitable packages; e.g. in rounds 11-15 in the CCA6 auctions, global bidders bid on only 2.0 out of 13.3 profitable packages available to bid on.[19] Further, we are unable to reject a null hypothesis in any of the cells that the number of packages bid on is the same between Easy

[18] These percentages are independent of each other in that a bid on the second most profitable package is counted regardless of whether or not a bid was placed on the most profitable package.
[19] In repeated canvassing, the subjects indicated that they had enough time to bid on all the packages they wanted, so this finding is not driven by time limits in the auction.

versus Hard auctions. Nor can we reject a null hypothesis that subjects direct their bids to the most profitable and second most profitable packages with equal frequencies in Hard versus Easy cases.[20]

To summarize: (i) players bid on only a small percentage of the profitable packages in each round and omit some packages entirely from their bidding during the auction, (ii) bids are largely directed at the most profitable packages, and (iii) we are unable to reject a null hypothesis that the number of packages bid on and the frequency of bidding on the most profitable package are the same between Hard versus Easy auctions.

If CCA prices fail to guide effective bidding and bidders bid on only few packages in each round, our theoretical conditions could still be satisfied if bidders bid sufficiently aggressively on all of their packages at appropriate times during the auction. But this is far from what we observe. A Global bidder on average bids at least once on only 6.0 distinct packages out of the 15 packages they could bid on during the course of a CCA4 auction. In other words, on average 9.0 packages never receive any bid at all from the Global bidder during the auction (and hence correspond to a reported value of zero). Regional bidders come closer to the necessary requirement: on average, they bid at least once during the auction on 2.1 distinct packages out of 3 packages. For CCA6, the averages are 17.6 out of 63 packages for the Global bidders and 5.1 out of 7 packages for the Regional bidders.

Second, most losing bidders in the auction had fully exhausted their profit opportunities on their selected packages by the last bidding round. This behavior is part of the sufficient conditions for both Theorems 1 and 2.

Table 3 reports the scope for potential profits available at the end of the auction, distinguishing between losing and winning bidders. The most notable element here is the difference between regional and global bidders in the frequency with which *losing* bidders could have possibly obtained higher positive profits by continuing to bid, averaging 6.7% for global bidders versus 25.0% for regional bidders. These differences, which are statistically significant at the 1% level in a random-effects probit (controlling for repeated measures for the same subject) suggest a *threshold problem*: one, or both, of the regional bidders bids less aggressively, hoping

[20] These statistical tests involved two-tailed ($p = 0.05$) non-parametric Wilcoxon signed-rank tests with average subject data as the unit of observation. The data corresponding to Table 2 broken out by Hard versus Easy cases is reported in the online appendix http://www.econ.ohio-state.edu/kagel/KLM2010AppendixOnline.pdf.

that the other will cause prices to increase sufficiently to defeat the global bidder. The magnitude of the foregone profits was not that large averaging 14.9 and 32.1 ECUs in CCA4 and CCA6 auctions, respectively.

[Insert Table 3 here]

There are no comparable differences between global and regional bidders when they were winning with respect to foregone (potential) profits. This reinforces the notion of a threshold problem for losing bidders.

Our third observation is that bidders tend to bid much more often on their most profitable package than on their less profitable packages. Note, however, that the mere fact that the most profitable packages attract the most bids cannot prove that bidders are guided primarily by prices and profits. The same packages might be selected by other criteria. In many cases, particularly early on, the most profitable package and the "named" package coincide. The latter is the package of all items for the global bidder and the package of all positively valued items for the regional bidders. To establish the degree to which prices and profits guide bidding, we focus on those cases where named packages were *different* from the most profitable ones. Table 4 reports these data for the regional bidders. As shown, when there was a conflict between the named package and the most profitable package, and bidders chose to bid on only one of the two, the most profitable package attracts more attention from regional bidders in all rounds, often by a wide margin. Note, however, that the named package or the named package *and* the most profitable package together still attract a reasonable percentage of bids, which as shown below, has consequences for the actual data versus the simulation model.

Table 4 leaves out data for global bidders, as there were very few cases (2 for CCA4; 12 for CCA6) where the named package was *not* the most profitable package in a given round. Averaging over these few cases, only the most profitable package was bid on 42.8% of the time, with only the named package bid on 14.3% of the time and both of them bid on 14.3% of the time, a pattern not unlike that reported for the regional bidders.

[Insert Table 4 here]

Finally, subjects typically did *not* place bids in rounds in which they were provisional winners. This effect was most pronounced in later rounds when the auction had a greater chance of ending immediately. In auction rounds 11 and above, global (regional) bidders failed to submit new bids in 95.9% (89.2%) of all rounds in which they were provisional winners in CCA4 auctions, and in 90.6% (87.3%) for the CCA6 auctions.[21] The reasons for these high frequencies are threefold: (i) subjects do not bid in every round even when they are *not* provisional winners (see below), (ii) bidding on packages as a provisional winner can extend the auction and/or raise prices on provisionally winning bids with unknown consequences, so that provisional winners were willing to settle for what they already had, and (iii) given the bid patterns, more often than not the profit on the provisionally winning package was greater than or equal to the potential profit from any new package that could be bid on.

On this last point, in rounds 11 and higher, provisionally winning bidders rarely bid in a round in which their provisional profit was higher than they could earn on any new bid, with no new bids in 98.9% (95.5%) of all such cases for global (regional) bidders in the CCA4 auctions and for 96.8% (88.7%) of all such cases for global (regional) bidders in the CCA6 auctions. Provisional winners stood pat somewhat less often when there was greater potential profit to be had on another package, with no new bids in 82.1% (57.9%) of all such cases for global (regional) bidders in the CCA4 auctions and 61.8% (76.9%) of all cases for global (regional) bidders in the CCA6 auctions. Bidders were substantially more likely to bid following a round in which they had not secured a provisionally winning bid, bidding in 70.4% (60.7%) of all such cases in the CCA4 and in 75.7% (73.8%) of all cases for global (regional) bidders for CCA6 auctions. Finally, looking at those cases in which a provisionally winning bidder did not bid and was not winning on her most profitable package, the profit difference compared to their best alternative averaged 31.1 (13.2) ECUs for global (regional) bidders in the CCA4 auctions, and 54.7 (13.5) ECUs in the CCA6 auctions.

Efficiency

Efficiency is calculated as $\left(S_{actual} - S_{random}\right)/\left(S_{max} - S_{random}\right)$, where S_{actual} is the actual realized surplus from the auction, S_{random} is the mean surplus resulting from a random allocation, and S_{max} is the maximum possible surplus.[22] With this measure, in every environment, the mean

[21] For rounds 1-10, the corresponding percentages are 81.1% and 88.0% for global and regional bidders in CCA4 auctions and 63.6% and 71.1% for global and regional bidders in CCA6 auctions, respectively.
[22] The value of the random allocation is computed by taking the average of the surplus over all possible allocations – 3^4 and 3^6 respectively – assuming all items are sold in each auction.

efficiency of a random assignment of goods is 0% and the efficiency of a surplus-maximizing assignment is 100%.

Table 5 reports efficiency for CCA and SAA auctions with four and six items, categorizing the results into the previously defined Easy (where the simulations achieve 100% efficiency) and Hard cases (all other cases). Average efficiency is significantly higher in Easy compared to Hard CCA auctions, for both 4-item and 6-item cases ($p < 0.01$).[23] More dramatic yet are the differences in the frequency with which Easy CCA auctions achieve 100% efficiency, averaging over 80% versus 40% or less for Hard CCA auctions ($p < 0.01$for both cases).

[Insert Table 5 here]

These efficiency differences between Easy versus Hard support hypothesis 2, as the data summarized in the previous section are at least qualitatively consistent with the assumptions used in the simulations: Subjects bid on only a limited number of packages often including the currently most profitable ones. This suggests that the simulator may work more generally to distinguish hard versus easy environments for the CCA to achieve efficient outcomes.

As noted above, in our experiment bidders do not rely solely on profits to decide which packages to bid on (recall Table 4). Bidders also bid on the named packages, even when they are not the most profitable ones. In the following we show how this behavior affects the efficiency of outcomes.

A closer look at auctions within the Hard category show important and systematic differences. Although all the auctions in the Hard category fail to achieve 100% efficiency in the simulations, for a number of these auctions the named packages correspond to the relevant packages, thereby attracting some attention even when they were not the most profitable packages.[24] In what follows we will refer to these as Medium Hard auctions. For the remaining auctions in the Hard category, the relevant packages in terms of hypothesis 2 either involve all bidders getting one or more items, or splitting items between one of the regional bidders and the

[23] Statistical tests comparing between CCA auctions in Tables 5, 6 and 8 are all one-tailed as they are based on simulated outcomes with clear predictions. Comparisons between CCA and SAA auctions in these tables are all two-tailed. Non-parametric Mann-Whitney tests continue to be used when conducting pair-wise tests for differences in average efficiency between cases. Binomial tests are used when conducting pair-wise tests for differences in frequencies of achieving 100% efficiency between cases.

[24] In the Easy auctions all of the relevant packages are named packages as well as being the most profitable packages.

global bidder. This second subset of auctions, which we will refer to as Very Hard, had lower average efficiency than the Medium Hard auctions in the simulations.[25]

[Insert Table 6]

Table 6 reports efficiencies for the Medium Hard and Very Hard auctions, as well as repeating the efficiency numbers for the Easy auctions from Table 5. First, notice that average efficiency is substantially lower in the Very Hard CCA auctions compared to either the Easy or the Medium Hard CCA auctions, as is the frequency with which these auctions achieve 100% efficiency ($p < 0.01$ in all cases). The Medium Hard CCA auctions are much more comparable in terms of average efficiency to the Easy CCA auctions than to the Very Hard CCA auctions. They achieve slightly higher average efficiency in the Easy CCA4 auctions and slightly lower average efficiency in the Easy CCA6 auctions.[26] Medium Hard CCA auctions achieve 100% efficiency less often than the Easy CCA auctions, with the difference significant at the 5% level for CCA6, and just missing statistical significance ($p = 0.102$) for CCA4.[27]

Efficiency in the CCA auctions is not uniformly higher than in the SAA auctions. In the Easy category, the CCA auctions yield significantly higher efficiency than the corresponding SAA auctions ($p < 0.01$), averaging a little over 10% higher efficiency, and the Easy CCA auctions achieve 100% efficiency substantially more often than in the corresponding SAA auctions ($p < 0.01$). But in the Hard category, average efficiency in the CCA auctions is not significantly different from the SAA auctions ($p > 0.20$). For Very Hard cases, the CCA had lower average efficiency, and substantially lower frequencies of 100% efficiency, than the corresponding SAA auctions for both the 4 and 6-item auctions ($p < 0.05$ in all cases).

These findings provide evidence that the relative efficiency of the CCA and SAA auction mechanisms depend on the parameter configurations. Further, the similarity in outcomes between the Easy and Medium Hard CCA auctions serve to contradict any hypothesis that behavior in our experiment is entirely guided by price and profits (as it is in our simulation).

[25] For the 6-item case average efficiency for the Very Hard auctions was 90.6% in the simulations with only one of six auctions having average efficiency over 93% whereas efficiency averaged 95.1% for the Medium Hard auctions with only one of six having efficiency lower than 93%. In the 4-item case efficiency averaged 87.5% for the Very Hard auctions versus 93.6% for the Medium Hard case, with two (out of six) of the Very Hard auctions having higher efficiency than the two Medium Hard auctions.
[26] This difference is statistically significant at the 10% level using a one-tailed Mann-Whitney test for the CCA6 auctions. But not significant at conventional levels for the CCA4 case.
[27] One-tailed binomial tests in both cases.

Rather, the data indicate that there is little difference in outcome efficiency between the Easy and Medium Hard cases, suggesting that named package play an important role in guiding bidding. Very Hard auctions provide further evidence consistent with this explanation, as bidding on the single most profitable package or the named package does not consistently point bidders to the efficient outcome in these cases. This is a reminder, if any is needed, about the limits of simulators and the importance of identifying general properties of actual bidder behavior when attempting to predict auction outcomes.

Seller Revenues

For a competitive-revenue standard for package auctions, we follow Milgrom (2007) in using the minimum seller revenue at any core allocation. The core for package allocation problems has a competitive auction interpretation: an individually rational allocation is in the core if there is no group of bidders who could all do better for themselves and for the seller by raising some of their losing bids. To enhance comparability, we report revenue in each auction as a percentage of the minimum revenue in the core. The experimental outcomes are reported in Table 7.

Our main hypotheses do not concern levels of revenues and profits. Compared to efficiencies, which are about allocations, revenue and profits may depend more sensitively on bidder behavior and the naïve bidding behavior used in the simulation may not be sufficiently descriptive to predict these outcomes. Also, the parameter selection procedure was based on outcome efficiencies, which is not the most suitable choice when evaluating how environments affect revenues and profits. We nevertheless report outcomes based on the Hard and Easy cases.

[Insert Table 7 here]

We find no evidence that the revenues in the CCA differ significantly between the Hard and Easy categories ($p > 0.50$).[28] Thus, the factors that lead to high efficiencies may differ from those that lead to high revenues in CCA. Comparing CCA with SAA, the only significant difference identified is that revenue in the SAA4 Hard auctions is significantly higher ($p < 0.01$) than in the CCA4 Hard auctions. Note that in this one case average revenue in the SAA auctions is over 100% of minimum revenue in the core. We will return to this point in the next section where profits are discussed.

[28] Statistical tests for Table 7 are all two-tailed Mann-Whitney tests using each auction as the unit of observation.

There are no directly equivalent revenue results from other multi-unit auction experiments. Porter et al. (2003) do not report revenue comparisons between auction mechanisms. Brunner et al.'s (2007) normalization reports revenue as a percentage of the efficient allocation.[29] They find that revenue is significantly higher in their version of the CCA auction than the simultaneous multi-round (SMR) auction employed by the FCC (the closest relative to our SAA auction). This contrasts with our results as revenues are not significantly different in the six-item case and are significantly higher in the SAA in the four-item Hard case. Their auctions involve bidding over more items than ours and include two global bidders competing over the same set of licenses. Also, because their mechanism withholds information about provisionally winning bids, it may encourage more bidding. Their revenue results hold for both high- and low-synergy cases. However, this comparison is strained by the fact that they had a relatively large number of items left unsold in their SMR auctions.

Bidder Profits

Table 7 reports profits as a percentage of the efficient allocation. As between Hard and Easy CCA auctions, total profit is significantly higher in the Easy CCA6 auctions ($p < 0.01$), with most of the difference being accounted for by the higher profits for regional bidders in the Easy auctions ($p < 0.01$). Comparing between CCA and SAA auctions, total profits are markedly higher in the CCA auctions for the Easy and Hard four-item auctions as well as the Easy six-item auctions ($p < 0.05$ in all cases). For the remaining Hard six-item case there are no significant differences between CCA and SAA auctions, with total profits somewhat higher in the SSA auctions for the Hard case (but $p > 0.10$). Note that in each of the three cases where the CCA auctions generate higher profits, global bidders earn negative average profits in the SAA auctions, while the regional bidders earn positive average profits. These negative profits are reflective of the exposure problem which is eliminated in the CCA auctions and which is more severe for global as opposed to regional bidders within the SAA auctions. Further, it is these losses that contribute to revenue exceeding 100% of minimum revenue in the core for Hard auctions in the SAA4 auctions mentioned earlier.

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[29] Brunner et al. use actual revenue less the revenue from a random allocation in which bidders pay full value in the numerator and revenue from the efficient allocation less the revenue from a random allocation in the denominator, so that the difference lies in taking differences from average revenue resulting from a random allocation in both the numerator and denominator.

Our profit results stand in marked contrast to those reported in Brunner et al. (2007), where total bidder profits are lower, sometimes substantially lower, in the CCA compared to the SMR auctions. Differently from Brunner et al., we announced provisional winners following each round of bidding. Failure to reveal provisional winners may cause bidders to raise their own winning bids at least some of the time. In our design bidders who do not raise their own provisionally winning bids could still "bid against themselves" by placing a bid that overlaps with their provisionally winning bid. However, this is unlikely to occur in later auction rounds when final prices are set.

Distance from the Core

Table 8 reports distances from the core for both experimental outcomes and simulated auction outcomes, using a revised definition of Easy versus Hard. Our original definitions of Easy versus Hard were based on evaluating the efficiency of simulation outcomes, and easy cases for efficiency may not also be ones in which the simulated behavior lies close to the core. Nevertheless, to the extent that the simulator reflects actual behavior, it may also be helpful in making predictions about the distance of outcomes from the core. We use the simulator to create another classification based in part on distance from the core.

Auctions are categorized as Core-Easy if, in the simulations, they achieve 100% efficiency and the scaled distance from the core is no greater than 15%.[30] All other auctions, whether they achieve 100% efficiency or not, are classified as Core-Hard. Raw distance from the core is defined as the maximum violation of one of the inequalities defining the core. The scaled distance is the raw distance divided by the difference between full efficiency and efficiency resulting from randomly allocating items among bidders.

For the simulated outcomes, the average scaled distance from the core is 5% or less for the Core-Easy CCA auctions versus 24% or more for the Core-Hard ones, with corresponding differences in the fraction of simulated outcomes achieving zero distance from the core. The experimental outcomes do not achieve the same magnitude of difference between Core-Easy and Core-Hard as the simulations, but the differences are in the right direction for both the four- and six-item cases. This difference is statistically significant at the 1% level in the CCA4 auctions.

[30] Results are robust to alternative definitions, such as the simulated distances from the core being no greater than 10%. None of the auctions achieving 100% efficiency also achieve zero distance from the core. Employing a criterion of simulated distance from the core being no greater than 5% yields 2 CCA4 Core-Easy auctions and 1 CCA6 Core-Easy auction, too small a sample for meaningful results.

The difference between Core-Easy and Core-Hard is not as large in the CCA6 auctions, with average distance from the core significant at the 10% level, and percent of auctions with zero distance from the core significant at the 5% level.

[Insert Table 8 here]

Although our categorization of Core-Easy versus Core-Hard in terms of the simulations qualitatively predicts the experimental outcomes, it does not predict the levels accurately. Incorporating more details of bidder behavior into the simulation could narrow the discrepancy. But fine-tuning the bidder behavior model is not the main purpose of this paper.

Rounds to Auction Completion

The average number of rounds to completion was quite similar across auction mechanisms. For the auctions in the Easy category, CCA6 (CCA4) auctions required an average of 15.5 (16.7) rounds per auction versus 18.5 (15.4) rounds for the SAA6 (SAA4) auctions. For the Hard category, CCA6 (CCA4) auctions required an average of 17.5 (17.8) rounds per auction versus 18.0 (15.2) rounds for the SAA6 (SAA4) auctions. None of these differences is statistically significant at conventional levels.

The one thing that does stand out in the data is that, not surprisingly, total bidder profits decrease systematically as the number of rounds in a given auction increase, regardless of which auction mechanism is used. For example, average profits of provisionally winning bidders decreased monotonically over rounds 1-5, 6-10, 11-15, 15-20 and 20 or greater in the CCA6 auctions, going from a high of 208.4 ECUs in rounds 1-5 to a low of 17.6 ECUs in rounds greater than 20 for global bidders, and from a high of 113.6 ECUs to a low of 22.9 ECUs for regional bidders.

V. Conclusions

We employ theory and experiment to investigate when "standard" package auctions – and particularly the combinatorial clock auction (CCA) – lead to efficient or, more strongly, core outcomes in package allocation problems. Our analysis begins with two theorems asserting that if winning bidders bid most aggressively on their *efficiency-relevant* or *core-relevant* packages, and if losing bidders exhaust their profit opportunities, then the result is an efficient or core outcome, respectively. Our experiment uses the CCA and a matched simultaneous ascending auction (SAA).

In principle, differences in performance among auction environments could depend on how many packages subjects bid on in different situations, so we begin by looking for that. We find no evidence that these numbers vary between the Easy and Hard auction environments. Instead, bidders in our experiment typically bid on just the one or two most profitable packages and those packages often remained unchanged for many rounds during an auction. In our data, consistent with our theory, the CCA yields efficient allocations and core-level revenues most frequently when the packages that are selected by this sort of behavior are the relevant ones. We verify this by comparing the outcomes of our experiment to those of simple simulations, in which automated bidders bid only for the *single* most profitable package at each round.

The outcomes from our experiment are largely consistent with the two hypotheses we formulated in Section II: Subjects bid for many fewer packages than the profitable ones at the current round prices, and simulations based on such bidder behavior qualitatively help predict the efficiency of outcomes. Simulations were less effective, however, at predicting the distance of experimental outcomes from the core.

Our finding that price-guided auctions can fail to direct bidders to relevant packages early enough in the auction suggests possible improvements to the auction design. This failure could be greater in environments in which, unlike most of our auctions, bidders may have many core-relevant packages. One possible refinement is to make relevant bids more likely by making it easier to bid on more sets of licenses. That might be accomplished by implementing a richer bidding language than the XOR language of our experiment.

A second element in our CCA design inhibiting bidding on the relevant packages is the reporting of provisionally winning bids. This impacts outcomes in two ways: (1) reporting provisionally winning bids may help bidders to tacitly collude, stopping bidding early on if all bidders are satisfied with their current profits and (2) reporting provisionally winning bids encourages bidders to adopt a wait-and-see policy, hoping that a provisionally winning bid with a large profit margin will eventually become a winner particularly in the later rounds of the auction. There was at least one clear case of tacit collusion with bidding ending in round 3, with prices at their starting values and substantial profits for all bidders.[31] One way to control for such implicit collusion is to employ a tie-breaking rule which allocates packages to the smallest possible number of bidders, instead of randomly as in our experimental design. This would maximize the number of bidders without provisionally winning bids in early rounds thereby promoting defection from such tacit collusion. The provisionally winning bidders' wait-and-see policy can generate inefficient assignments and/or non-core (low revenue) outcomes. This happens because such bidders may fail to bid on alternative packages with greater potential profits and may miss opportunities to place higher bids on the same package. Withholding information about provisionally winning bids would eliminate the wait-and-see motive but would add other difficult strategic choices.

As emphasized in the introduction, the set of package auction environments is far too large to be convincingly explored with experiments alone. The simulation approach provides a predictive theory that offers the promise of generalizable conclusions. In our experiment, simulations, which make specific predictions about efficiency, revenue and the core, had some predictive success. We believe that simulations are a promising tool for testing and designing practical auction mechanisms.

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[31] In this case no new bids were placed after round 1. This happened mid-way through the second full session, so that subjects would have correctly anticipated that they could not do much better by continuing to compete. Profits were 45, 26, and 43 ECUs for the two regional bidders and the global bidder, respectively.

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Table 1
Experimental Treatments

Session	**Number of Subjects**[a] Session 1[b]	Session 2	Session 3
Combinatorial Clock Auction (CCA)			
4 items	22	20	18
	(3)	(9)	(8)
6 items	19	18	16
	(3)	(9)	(10)
Simultaneous Ascending Auction (SAA)			
4 items	21	20	19
	(3)	(9)	(8)
6 items	21	21	19
	(3)	(9)	(10)

[a] Number of auction valuations in each session in parentheses. Same subjects participated in a given series (e.g., CCA with 4 items). Number of subjects decreases over sessions due to attrition.

[b] Data from Sessions 1, the training session, are not included in the analysis.

Table 2
Packages Bid on in CCA Auctions[a]

	Global Bidders			Regional Bidders[b]		
		Distribution of Bids[d]			Distribution of Bids[d]	
CCA 4 Auctions	Average Number of Bids[c]	Most Profitable	2nd Most Profitable	Average Number of Bids[c]	Most Profitable	2nd Most Profitable
Rounds 1-5	3.9 (13.8)	76.7%	43.9%	1.5 (2.8)	91.0%	44.4%
Rounds 6-10	1.6 (7.4)	81.7%	16.6%	1.2 (2.3)	84.7%	35.3%
Rounds 11-15	1.3 (4.2)	86.6%	19.4%	1.1 (2.1)	87.5%	26.3%
Rounds >15	1.2 (3.6)	87.5%	12.5%	1.1 (1.8)	89.1%	15.5%
CCA 6 Auctions						
Rounds 1-5	11.5 (60.5)	79.0%	55.4%	3.1 (6.8)	81.1%	54.6%
Rounds 6-10	2.6 (35.7)	72.9%	32.4%	1.9 (5.5)	77.7%	45.6%
Rounds 11-15	2.0 (13.3)	85.2%	27.2%	1.5 (3.9)	78.6%	40.2%
Rounds >15	1.3 (6.9)	86.8%	13.9%	1.3 (3.2)	93.1%	26.3%

[a] Data are only included for rounds in which a bidder is not a provisional winner, had at least one profitable package to bids on, and a bid was submitted.
[b] Only includes packages where all items had positive value for regional bidders.
[c] In parenthesis are average number of profitable packages available to bid on.
[d] Percentages add up to more than 100% as subjects bid on the most profitable package as well as the second most profitable package.

Table 3
Scope for Increased Profit at End of Auctions

	Bidder Type	Frequency Higher Profits were Available[a]	Average Foregone (Potential) Profits in ECUs[b]
CCA4 Auctions			
Losing Bidders	Global	4.5% (3/66)	12.0 (7.6)
	Regional	25.5% (12/47)	14.9 (15.6)
Winning Bidders	Global	16.7% (6/36)	61.2 (51.8)
	Regional	6.4% (10/157)	20.3 (29.8)
CCA6 Auctions			
Losing Bidders	Global	9.4% (5/53)	45.2 (40.7)
	Regional	24.7% (19/77)	32.1 (9.5)
Winning Bidders	Global	9.8% (5/51)	52.4 (16.4)
	Regional	9.9% (13/131)	23.7 (7.7)

[a] Raw data in parentheses.

[b] Standard error of the mean in parentheses.

Table 4
Package Bids in CCA Auctions when Named Package No Longer the Most Profitable Bid[a]

		Regional Bidders		
CCA 4 Auctions	(number cases)	Most Profitable Only	Named Package Only	Both Most Profitable and Named
Rounds 1-5	(16)	43.8%	6.3%	18.8%
Rounds 6-10	(126)	42.1%	9.5%	14.3%
Rounds 11-15	(98)	50.0%	9.2%	10.4%
Rounds 16-20	(35)	40.0%	11.4%	5.7%
Rounds > 20	(12)	41.7%	16.7%	0.0%
CCA 6 Auctions				
Rounds 1-5	(7)	57.1%	0.0%	14.3%
Rounds 6-10	(105)	27.6%	12.4%	20.0%
Rounds 11-15	(92)	26.1%	9.8%	10.9%
Rounds 16-20	(12)	58.3%	8.3%	8.3%
Rounds > 20	(9)	33.3%	22.2%	11.1%

[a] Observations for which named package is not profitable are dropped. When a provisional winner does not bid, or the auction is in the last round, observations are dropped.

Table 5
Outcomes in Easy vs. Hard Package Auction Outcomes
(standard error of the mean in parentheses)

		CCA Efficiency		SAA Efficiency	
		Average	Percent of Auctions Achieving 100% Efficiency	Average	Percent of Auctions Achieving 100% Efficiency
4-item auctions	Easy (9)[a]	95.7% (2.0)	88.9%	82.2% (3.1)	50.0%
	Hard (8)[a]	91.5% (1.6)	35.4%	92.9% (1.6)	50.0%
6-item auctions	Easy (7)[a]	95.3% (1.8)	82.1%	83.9% (2.0)	28.3%
	Hard (12)[a]	92.4% (1.2)	40.0%	90.9% (1.1)	32.5%

[a] Number of value profiles

Table 6
Outcomes in Medium Hard vs. Very Hard Package Auctions
(standard error of the mean in parentheses)

		CCA Efficiency		SAA Efficiency	
		Average	Percent of Auctions Achieving 100% Efficiency	Average	Percent of Auctions Achieving 100% Efficiency
4-item auctions	Easy (9)[a]	95.7% (2.0)	88.9%	82.2% (3.1)	50.0%
	Medium Hard (2)[a]	96.4% (2.2)	75.0%	92.6% (2.1)	25.0%
	Very Hard (6)[a]	89.9% (1.9)	22.2%	93.0% (2.0)	58.3%
6-item auctions	Easy (7)[a]	95.3% (1.8)	82.1%	83.9% (2.0)	28.3%
	Medium Hard (6)[a]	94.2% (1.6)	59.4%	87.8% (1.3)	15.8%
	Very Hard (6)[a]	90.7% (1.7)	21.2%	93.8% (1.5)	48.7%

[a] Number of value profiles

Table 7
Revenue and Profits in CCA and SAA Auctions
(standard error of the mean in parenthesis)

		Revenue[b]		Profit[c]		Global Profit[c]		Local Profit[c]	
		CCA	SAA	CCA	SAA	CCA	SAA	CCA	SAA
4 –item auctions	Easy[a] (9)	92.1% (2.0)	98.6% (2.4)	21.4% (1.2)	8.8% (2.7)	5.1% (1.2)	-3.4% (1.5)	8.2% (0.8)	6.1% (0.9)
	Hard[a] (8)	92.9% (2.4)	102.2% (2.7)	20.1% (1.8)	12.4% (2.7)	3.4% (0.9)	-2.6% (1.4)	8.4% (0.9)	7.5% (1.0)
6 – item auctions	Easy[a] (7)	90.4% (3.1)	94.2% (3.0)	18.5% (2.4)	8.7% (2.6)	4.6% (2.1)	-0.8% (1.4)	6.9% (1.0)	4.7% (1.0)
	Hard[a] (12)	92.9% (1.4)	86.7% (2.5)	11.9% (1.2)	16.2% (2.3)	4.2% (1.0)	4.5% (1.1)	3.8% (0.5)	5.9% (0.9)

[a] Easy versus Hard categories defined as in Table 5 above. Number of value profiles in parentheses.
[b] Measured as a percentage of minimum revenue in the core.
[c] Measures as a percentage of the efficient allocation

Table 8
Simulated Outcomes versus Experimental Outcomes: Distance from the Core

		Auction Outcomes[b]		Simulation Outcomes[c]	
		Average Distance	Percent of Auctions Achieving Zero Distance	Average Distance	Percent of Auctions Achieving Zero Distance
4-item auctions	Core-Easy (8)[a]	14.0% (3.3)	18.8%	4.6% (5.0)	44.3%
	Core-Hard (9)[a]	21.9% (2.4)	3.7%	29.7% (20.4)	5.3%
6-item auctions	Core-Easy (5)[a]	18.6% (4.9)	10.7%	5.0% (5.1)	39.7%
	Core-Hard (14)[a]	20.5% (2.2)	2.6%	24.9% (14.9)	1.8%

[a] Number of value profiles
[b] Standard error of the mean in parentheses
[c] Standard error in parentheses