Econometrica, Vol. 49, No. 4 (July, 1981)

RATIONAL EXPECTATIONS, INFORMATION ACQUISITION, AND COMPETITIVE BIDDING

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Most rational expectations market equilibrium models are not models of price formation, and naive mechanisms leading to such equilibria can be severely manipulable. In this paper, a bidding model is developed which has the market-like features that bidders act as price takers and that prices convey information. Higher equilibrium prices convey more favorable information about the quality of the objects being sold than do lower prices. Bidders can benefit from trading only if they have a transactions motive or if they have access to inside information. Apart from exceptional cases, prices are *not* fully revealing. A two stage model is developed in which bidders may acquire information at a cost before bidding and for which the equilibrium price *is* fully revealing, resolving a well-known paradox.

1. INTRODUCTION

Since the introduction in the early 1950's of the Arrow-Debreu theory of general equilibrium under conditions of certainty, a large literature has evolved seeking to extend that theory to accommodate production and trade under uncertainty. A central feature of the Arrow-Debreu theory is that commodities are identified by their attributes, including both their physical characteristics and such factors as the time and place at which the commodity becomes available. To accommodate uncertainty, Debreu [3] suggested viewing the contingencies (or "states of the world") in which a commodity is deliverable as an attribute, and Arrow [1] showed how this contingent commodity perspective leads to a theory of securities as instruments for distributing risk. Radner [24] added an explicit formulation of the information available to traders in the Arrow-Debreu model.

When each trader is endowed with his own private source of information, or when traders can acquire information at a cost, the traders' strategic options may be drastically different than in the case where all information is public. It may be possible, for example, for a trader to infer information from the terms of trade he is offered or, more generally, from any observations he makes concerning the behavior of other traders. Some rational expectations equilibrium (REE) models attempt to capture this process of inference. I argue below that the existing REE models are defective in important ways, and I offer a partial alternative which escapes these defects.

For the purposes of this discussion, a *state of information* is a list X whose ith entry X_i describes the private information of trader i. A *rational expectations*

[1] I gratefully acknowledge the insightful suggestions of my friend and colleague Rick Antle and the guidance of my dissertation adviser, Robert Wilson. John Geanakoplos discovered an error in an earlier version, and enhanced my understanding of the role of ties. D. John Roberts helped me with the exposition. Bengt Holmström, Roger Myerson, Robert Weber, and two anonymous referees (including one from another journal) offered helpful comments on an earlier draft. This work was partially supported by the Center for Advanced Study in Managerial Economics at Northwestern University and by NSF grant SES-8001932 to Northwestern University.

The first version of this paper was titled "A Bidding Model of Price Formation Under Uncertainty."

equilibrium is a pair of functions mapping states of information into a price vector and an allocation, respectively, with the properties that: (i) each trader maximizes his expected utility, subject to his budget constraint, (ii) the net trades sum to zero, and (iii) each trader's expectations properly reflect both his private information and any information which can be inferred from the vector of prices. Traders in these models are assumed to act as price takers in a new and expanded sense; they believe that their actions will affect neither the terms of trade nor the informational content of prices.

A recurrent idea in REE models is the idea that prices are *fully revealing*, i.e., that the price vector is a sufficient statistic for all of the information observed by all of the traders. Let $d_i(X_i, p)$ be the net trade demanded by trader i in some fully revealing REE when he observes X_i and prices are p. Since prices are fully revealing, the trader can ignore his private information in forming his demands, so that d_i does not actually vary with X_i. (An example of this kind has been given by Grossman **[9]** and this point is elaborated by Beja **[2]**.) Let f be the equilibrium function mapping states of information into price vectors. Then, for every state of information X, markets clear: $\sum d_i(X_i, f[X]) \equiv 0$. Let p^* be the equilibrium price vector corresponding to some state of information X^*: $p^* = f(X^*)$. Then $\sum d_i(X_i^*, p^*) = 0$. But net demands do not depend on the traders' private information, so for every X, $\sum d_i(X_i, p^*) = 0$. In other words, any price in the range of f clears the markets in every state of information X![2]

The problem here arises from the expanded definition of price taking behavior. Traders ignore their information because they see it reflected in the prices. But how does private information come to be reflected in prices if no trader uses his information?

In the context of the Arrow-Debreu model, Roberts and Postlewaite **[27]** have justified the standard price taking behavior assumption for large exchange economies by showing that the incentive to deviate from price taking behavior is small and that individual traders can have little effect on prices. In the context of a fully revealing REE, even this standard price taking assumption is hard to justify.[3,4] Following Roberts' and Postlewaite's approach, suppose that there are

[2]This critique is most cogent for models like Grossman's **[9]** in which no trader can narrow the range of possible prices by referring to his private information. For such models, the equilibrium excess demand functions $\{d_i(X_i, p)\}$ *may not* depend on p. At the opposite extreme, there are REE models, such as those of Green **[8]** and Radner **[25]**, in which for each trader i the range of the equilibrium price function for a given realization x_i of X_i (i.e., $f(X \setminus x_i)$) is disjoint from the range given any other realization y_i. For such models the trader's equilibrium excess demand functions are not uniquely specified and may be chosen to depend on the private signals. This, however, merely buries the question one layer deeper. What is the trader to think if the announced price is inconsistent with his private information? The out-of-equilibrium process is simply not specified for these models.

[3]For the reasons cited in note 1, this argument applies with more force to models like Grossman's than to those like Green's or Radner's. These latter models make it harder to specify sensible out-of-equilibrium behavior, and therefore harder to criticize on grounds of stability and manipulability.

[4]Kobayashi **[16]** has devised a price formation process in which traders, acting as price takers, grow progressively more sophisticated. A rational expectations equilibrium emerges after a finite number of iterations in his model.

n traders and that the first $n-1$ traders adopt their equilibrium behavior, as described by the net demand functions $\{d_i\}$. Suppose trader n adopts a demand function d_n^* that agrees with d_n only at $p = p^*$. As argued above, p^* is then a market clearing price, but one can say more: it is the unique market clearing price. Indeed, at any other price, $\sum_{i=1}^{n-1} d_i(X_i, p) + d_n^*(X_n, p) \neq \sum_{i=1}^{n} d_i(X_i, p) = 0$. Thus, by the appropriate choice of d_n^*, trader n can achieve any net trade in the range of d_n!

Grossman and Stiglitz **[11]** have called attention to another difficulty in the REE paradigm. If information is costly and prices are fully revealing, and if the traders believe that they cannot affect the informational content of prices, then at equilibrium no trader pays to gather information because all of his information is already freely available to him in the prices. But if no trader gathers information, then the prices can convey no information. Finally, if information is sufficiently cheap and prices convey no information, then some trader will want to gather information. In short, the system may have no equilibrium. Grossman and Stiglitz conclude that fully revealing equilibrium prices are logically impossible, though the root cause of their paradox is probably the peculiar definition of price taking behavior. Note that each Grossman-Stiglitz trader can clearly see his own private information—information which is available to nobody else—reflected in the prices, yet he believes that he can have no effect on the prices.

To address seriously such questions as: (i) "How do prices come to reflect information?", (ii) "Are there incentives to deviate from price-taking behavior?", and (iii) "How do information gathering decisions affect prices?", one needs a theory describing how prices are formed. I have argued above that the REE paradigm (which is not a theory of price information) does not provide ready answers to these questions.

One class of price formulation mechanisms which is of great empirical significance and which has recently been the subject of intensive theoretical studies is the class of competitive auctions and bidding processes. Many of the papers in this area study models where the objects being traded are of known quality; only the preferences of opposing bidders are unknown. In a few models, however, the objects traded have unknown attributes and the several bidders have unequal access to information. The equilibria of these models have much the same flavor as rational expectations equilibria. For example, Wilson **[35]** and Milgrom **[19]** have studied models of a sealed-bid tender auction for a single indivisible object, with each bidder having only sample information about the object's value. As the number of bidders grows large, the winning bid may converge to the true value of the object, even though no bidder knows that value when the bids are tendered. Thus, the price (i.e., the winning bid) aggregates information from many bidders, and the bidders' equilibrium strategies are deeply influenced by this fact.

In this paper, I attempt to strengthen the link between the bidding and rational expectations literatures. I study a model in which k identical objects are offered for sale to n bidders ($n > k \geqslant 1$). Sealed bids are tendered, and the k highest bidders each receive one object for a price equal to the $k + 1$st highest bid. This is the "highest rejected bid mechanism," which was introduced by Vickrey [32].

Focusing attention on a single bidder i, let W_i be the kth highest bid among the other bidders. One may think of W_i as the price faced by bidder i. If he tenders a bid higher than his price, he acquires one object for that price. If he tenders a lower bid, he acquires nothing.

With this interpretation, each bidder is a "price taker"; he cannot individually affect his price. Although different bidders may face different prices, all trades take place at a single price.

Since the bids are submitted simultaneously, bidders must make their bids in ignorance of the prices. This feature distinguishes the bidding model from REE models. However, I shall prove that if a bidder were informed of his price and extracted all of the information which his price conveys, he could never gain by revising his bid. Thus, the bidders are not only price takers, they also compute their "demands" as if they had full access to price information.

In the context of this model, one can study a bidder's incentives for gathering private information. I shall show that a bidder without special private information and without a transactions motive for bidding can never earn a positive expected payoff. The best course for such a bidder is to withdraw from the auction, i.e., to bid zero.

To emphasize this zero-payoff result, an example is given in which prices are fully revealing, i.e., W_i is a sufficient statistic for the information of the bidders other than i. The zero payoff theorem implies that each bidder has an incentive to gather information, despite these fully revealing prices. Like the traders of Grossman and Stiglitz, the bidders in my model assume they cannot affect prices. However, unlike the traders' assumptions, the bidders' assumptions are fully justified. Thus, there is no tension in my model between the incentive to gather information and the informational efficiency of prices.

A desirable property of the bidding model is that it captures some of the details of real securities markets. In a typical small securities transaction for a listed security, the buyer places a limit order, i.e., he instructs his broker to obtain the most favorable possible terms, but not to pay more than some specified limit price. (This limit price corresponds to a bid in the model.) He expects to successfully acquire the security whenever his limit price is higher than the prevailing price in the market, and he expects that the limit price he names will not affect the price that he must pay. All of these expectations are justified within the bidding model.

This paper is arranged in seven sections, including the present one. In Section 2, I introduce the ideas of news and good news as tools in modeling information. Section 3 is devoted to stating the assumptions, developing the notation, and presenting the equilibrium. Various properties of the equilibrium, including the zero payoff result, are presented in Section 4. A two stage game in which traders must choose whether or not to gather costly information before bidding is analyzed in Section 5. Then, in Section 6, I study some variations of the basic model. The conclusion expresses my views concerning what can be learned from the analysis and what remains to be done.

2. NEWS AND GOOD NEWS

Let Z denote the unknown quality of the objects being sold at auction. I take Z to be a real-valued random variable, with larger values of Z corresponding to better quality. For example, if the objects sold are mineral rights on adjacent tracts of land, Z may denote the market value of the recoverable resources. If a single work of art is being sold, $Z = 1$ may indicate that the work is an original while $Z = 0$ may indicate a copy.

Depending on the particular application, information about Z can come in various forms, ranging from tables of statistical data to satellite photographs to consultants' reports. In general, one can represent bidder i's information by a random variable X_i taking values in some abstract measure space. The variable X_i will convey information as long as it is not independent of Z. Let $\mu_i(\cdot \mid Z)$ denote the conditional distribution of X_i given Z.

Whatever form information may take, I shall want to make comparisons between relatively good news and relatively bad news. Informally, a signal (or report, or piece of news) x is *more favorable than* y if for every nondegenerate prior distribution G on Z the posterior $G(\cdot \mid x)$ dominates $G(\cdot \mid y)$ in the sense of strict first order stochastic dominance; and x is *equivalent* to y if $G(\cdot \mid x)$ and $G(\cdot \mid y)$ are always identical. The variable X_i has the *signal ordering property* if for every pair of nonequivalent signals x and y in the range of X_i, either x is more favorable than y or y is more favorable than x.

There is a standard property of statistical distributions that is closely related to the signal ordering property. For simplicity, let Y be a real-valued random variable whose conditional density given some value z of Z is $f(\cdot \mid z)$. For this case of variables with densities, the monotone likelihood ratio property can be found in standard references (e.g., Ferguson [7]).

DEFINITION: Y has the (strict) *monotone likelihood ratio property* if the likelihood ratio function $f(y \mid z)/f(y \mid z')$ is nonincreasing (decreasing) in y (on its domain of definition) whenever $z' > z$ and nondecreasing (increasing) whenever $z' < z$.

The general definitions of the MLRP and of the signal ordering property require that attention be given to "versions" of densities, a measure-theoretic detail. (Throughout this paper, I shall always neglect such details.) A rigorous treatment of these properties has been given by Milgrom [**20**] and the following results have been proven.

THEOREM 2.1: *Y has the strict MLRP if and only if for every nondegenerate prior distribution G on Z and every y and y' in the range of Y with $y > y'$, $G(\cdot \mid Y = y)$ dominates $G(\cdot \mid Y = y')$ in the sense of (strict) first order stochastic dominance.*

THEOREM 2.2: *X has the signal ordering property if and only if there exists a real-valued function h on the range of X such that $h(X)$ is a sufficient statistic for X and has the strict MLRP.*

In view of Theorem 2.2, I shall henceforth simply assume that each bidder's signal variable X_i is a real-valued random variable with the strict MLRP. Thus, higher numerically valued signals will represent relatively better news than lower signals. Theorem 2.1 and the following two results will play important roles in the subsequent analysis.

THEOREM 2.3: *Let X have the MLRP and let $h:\mathbb{R}\to\mathbb{R}$ be increasing. Then $h(X)$ has the MLRP.*

PROOF: Simply apply Theorem 2.1 to X and $h(X)$. *Q.E.D.*

THEOREM 2.4: *Let $X_1, \ldots, X_n$ be random variables which, conditional on Z, are independent and identically distributed. Suppose that each X_i has the MLRP and let Y be the kth order statistic among the X_i's ($1 \leqslant k \leqslant n$). Then Y has the MLRP.*

PROOF: I shall treat only the case where the common distribution functions $F(\cdot\,|\,z)$ have given densities $f(\cdot\,|\,z)$. Then the density for the kth order statistic at x is:

$$\frac{n!}{(k-1)!\,(n-k)!}F^{n-k}(x\,|\,z)f(x\,|\,z)(1-F(x\,|\,z))^{k-1}. \tag{2.1}$$

Fix z' and z with $z' > z$. The likelihood ratio is then

$$\frac{F^{n-k}(x\,|\,z)}{F^{n-k}(x\,|\,z')}\;\frac{f(x\,|\,z)}{f(x\,|\,z')}\;\frac{(1-F(x\,|\,z))^{k-1}}{(1-F(x\,|\,z'))^{k-1}}. \tag{2.2}$$

The middle ratio of (2.2) is nonincreasing by the MLRP assumption, so the entire expression is nonincreasing if both $F(x\,|\,z)/F(x\,|\,z')$ and $(1-F(x\,|\,z))/(1-F(x\,|\,z'))$ are nonincreasing. Computing a derivative,

$$\frac{d}{dx}\frac{F(x\,|\,z)}{F(x\,|\,z')} = \frac{f(x\,|\,z)F(x\,|\,z') - f(x\,|\,z')F(x\,|\,z)}{F^2(x\,|\,z')} \tag{2.3}$$

which is nonpositive if $F(x\,|\,z)/F(x\,|\,z') \geqslant f(x\,|\,z)/f(x\,|\,z')$. By the MLRP,

$$\begin{aligned}\frac{F(x\,|\,z)}{F(x\,|\,z')} &= \int_{-\infty}^{x}\frac{f(s\,|\,z)}{f(s\,|\,z')}f(s\,|\,z')\,ds/F(x\,|\,z') \\ &\geqslant \int_{-\infty}^{x}\frac{f(x\,|\,z)}{f(x\,|\,z')}f(s\,|\,z')\,ds/F(x\,|\,z') \\ &= f(x\,|\,z)/f(x\,|\,z').\end{aligned} \tag{2.4}$$

This proves that $F(x\,|\,z)/F(x\,|\,z')$ is nonincreasing. The proof for the other term is similar. *Q.E.D.*

3. THE MODEL AND ITS EQUILIBRIUM

As indicated in the first two sections, I shall suppose that there are n bidders and k identical objects ($n > k \geqslant 1$) and that each bidder observes a real-valued random variable X_i whose conditional distributions have densities $f(\cdot \mid z)$ with the strict MLRP.[5] A bidder who receives no object is assumed to earn a payoff of zero. A bidder who pays b to receive one object earns $u(Z, X_i, b)$.

Earlier studies have focused on two special cases. First, when $u(Z, X_i, b) = X_i - b$, this model specializes to the one analyzed by Vickrey. The signal X_i then represents bidder i's personal valuation of the objects being sold. For this case, Vickrey showed that i's dominant strategy is to tender a bid equal to this valuation X_i.

The second special case arises when $u(Z, X_i, b) = Z - b$. The interpretation here is that each bidder would value the object at Z, if only he knew Z. Wilson [**34**, **35**] used such a model to analyze auctions conducted by the U.S. Department of the Interior for oil drilling rights. Similar models have been used by Rothkopf [**28**], Reece [**26**], Engelbrecht-Wiggans, Milgrom and Weber [**6**], Milgrom [**19**], and others.

For this analysis, I shall make the following assumptions.

ASSUMPTION A1—*Symmetry*: *Each player has identical prior beliefs represented by a distribution G on Z and conditional densities $f(\cdot \mid z)$ for X_i. The payoff functions u are identical across players.*

ASSUMPTION A2—*Conditional Independence*: *Conditional on Z, the signal variables $X_1, \dots, X_n$ are independent.*

ASSUMPTION A3—*Monotonicity*: *The payoff function $u(Z, X_i, b)$ is continuous and decreasing in b, nondecreasing in Z and X_i, and strictly increasing in either Z or X_i.*

ASSUMPTION A4—*Valuable Objects*: *For all z and x, $u(z, x, 0) \geqslant 0$.*

ASSUMPTION A5—*Valuable Money*: *For all z and x, there is some b such that $u(z, x, b) < 0$.*

ASSUMPTION A6—*Finite Expectations*: *For all b, $E[u(Z, X_i, b)]$ exists and is finite.*

Assumption A6 guarantees that all the expectations and conditional expectations used in this paper are finite. Although Assumption A3 is maintained

[5] The continuity of $F(\cdot \mid z)$ and the strictness of the MLRP work together to ensure that the probability of ties is zero in the equilibrium given in this section. Note 7 indicates how to compute and evaluate the equilibrium when one assumes only that signals have the MLRP, without assuming the strict MLRP or continuity of the signal distributions. This extension is an interesting one because it allows one to apply the model to examples where the signal distributions (given $Z = z$) are Poisson (with mean z), geometric (with mean z), or uniform (on $[0, z]$).

through the formal developments, I shall confine interpretive comments to the case where u is strictly increasing in Z.

The assumptions stated above are general ones by the standards of bidding models: bidders are allowed to be risk-averse or even risk-loving and preferences are not required to be additively separable in money. In Section 6, I further generalize the model by showing that Assumption A2 can be weakened to allow correlated information without weakening the substantive conclusions obtained from the model.

The situation described above can be regarded as a *Bayesian game*, as formalized by Harsanyi [**13**]. A *pure strategy* for bidder i is a function $p_i : \mathbb{R} \to \mathbb{R}_+$ taking signals into bids. Thus when bidder i observes X_i, he tenders the bid $p_i(X_i)$.

The bidders' payoffs in the game are determined as follows. Let W_i be the kth highest bid among the opponents of bidder i. The bid b *wins* for i if $b > W_i$. If $b = W_i$, the k highest bidders are not uniquely determined, and I assume that any ties are broken at random. Thus, if $b = W_i$, b may or may not win. If b wins (denoted $W_i \dot{<} b$), bidder i acquires one object for a price of W_i, so his payoff is $u(Z, X_i, W_i)$. If he submits a losing bid, his payoff is zero.

Holding the strategies p_j ($j \neq i$) fixed, bidder i may regard W_i as a random variable. His strategy p_i is called an *optimal response* to the opposing strategies if

$$(3.1) \qquad p_i(x) \in \arg\max_b E\left[u(Z, X_i, W_i)1_{\{W_i \dot{<} b\}} \mid X_i = x\right]$$

for all x in the range of X_i. (The notation "arg max" denotes the set of maximizers of the given expression.) An n-tuple of strategies $(p_1, \ldots, p_n)$ is an *equilibrium* if each strategy is an optimal response to the others.

The analysis of equilibrium in symmetric bidding models is usually guided by the "educated guess" that there will be a *symmetric* pure strategy equilibrium, that is, one for which $p_1 = \cdots = p_n \equiv p$, and that the equilibrium strategy will be increasing. With this guess in the background, define Y_i to be the kth order statistic among the variables $\{X_j \mid j \neq i\}$. If the guess made above is correct, then $W_i = p(Y_i)$.

Define the *reservation price function* g by:

$$(3.2) \qquad g(x, y) = \sup\{b \mid E[u(Z, X_i, b) \mid X_i = x, Y_i = y] \geqslant 0\}.$$

(In view of Assumptions A1 and A3–A6, this definition is meaningful.) If bidder i were able to observe $X_i = x$ and $Y_i = y$ and if no other information were available, then he would be willing to pay any price less than $g(x, y)$ to acquire one object, but he would be unwilling to pay any higher price.

THEOREM 3.1:[6] *The bidding game has a symmetric equilibrium and the equilib-*

[6]Equation (3.3) can be informally derived as the first order condition of a bidder's maximization problem if one guesses that there is a symmetric equilibrium with an increasing equilibrium bid function p. The first order condition then asserts that a bidder should be indifferent about whether he is selected as a winner when he is involved in a tie. This derivation appears in an earlier version of Matthews [**18**].

rium strategy is given by the increasing function:

$$p(x) = g(x,x). \tag{3.3}$$

Proof of Theorem 3.1 is postponed to the next section so that a relatively transparent graphical analysis can be used.

In the special case where $u(Z,X_i,b) = X_i - b$, it is easy to check that $g(x,x) = x$. This is the equilibrium discovered by Vickrey for his model. Vickrey's equilibrium is actually a dominant strategy equilibrium; a bidder can do no better than to use his equilibrium strategy regardless of the strategies adopted by the other bidders.

The second special case arises when $u(Z,X_i,b) = Z - b$. Then one can check that the reservation price function takes the following simple form:

$$g(x,y) = E[Z \mid X_i = x, Y_i = y]. \tag{3.4}$$

In Section 5, an example is given for which the equilibrium strategy is explicitly computed. The computation derives from the following expression, which in turn derives from (3.4) and Bayes Theorem.

$$g(x,y) = \frac{\int s f(x \mid s) f(y \mid s)(1 - F(y \mid s))^{k-1} F^{n-k-1}(y \mid s)\, dG(s)}{\int f(x \mid s) f(y \mid s)(1 - F(y \mid s))^{k-1} F^{n-k-1}(y \mid s)\, dG(s)}. \tag{3.5}$$

Before proceeding to study the properties of the equilibrium, let us establish some properties of the random variable Y_i and the functions g and p.

THEOREM 3.2: (i) *Y_i has the MLRP.* (ii) *$g(x,y)$ is increasing in x and nondecreasing in y.* (iii) *p is increasing.*

PROOF: That Y_i has the MLRP follows directly from Theorem 2.4. Since X_i also has the MLRP and since X_i and Y_i are conditionally independent, an application of Theorem 2.1 and Assumption A3 establishes that the conditional expectation

$$E[u(Z,X_i,b) \mid X_i = x, Y_i = y] \tag{3.6}$$

is increasing in x, nondecreasing in y, and decreasing and continuous in b. Then, from (3.2), it is routine to check that g has the properties claimed. Since $p(x) = g(x,x)$, (iii) follows directly from (ii). *Q.E.D.*

4. PROPERTIES OF THE EQUILIBRIUM

The analysis begins with a study of bidder i's problem when all opposing bidders adopt the strategy p. As shown below, i's price variable W_i then has the interesting and plausible property that higher prices convey more favorable news about Z than do lower prices, in the sense explained in Section 2.

THEOREM 4.1: *The variable W_i has the MLRP.*

PROOF: By Theorem 3.2, p is increasing, so

$$W_i = p(Y_i). \tag{4.1}$$

Apply Theorem 2.3. *Q.E.D.*

Theorem 4.1 raises a spectre that I have argued is a problem in rational expectations models. If bidder i could observe the price variable W_i before submitting his bid, his estimate of the value of the objects being sold would rise with W_i. This leaves open the possibility that i's value estimate would actually rise as fast or faster than prices do. To see how the bidding model avoids this difficulty, let us analyze the relationship between the price and i's valuation of the object given both his private information and the price information. (This valuation, of course, is a hypothetical one, since i does not actually observe price information before tendering his bid.) Define a function $\hat{g}$ on the product of the ranges of X_i and p by:

$$\hat{g}(x,w) = \sup\{b \mid E[u(Z,X_i,b) \mid X_i = x, W_i = w] \geqslant 0\}. \tag{4.2}$$

The functions g, $\hat{g}$, and p are related as follows:

$$g(x,y) = \hat{g}(x,p(y)). \tag{4.3}$$

THEOREM 4.2: *The function $\hat{g}(x,w)$ is increasing in x and nondecreasing in w. Moreover, the following relationships hold*:

$$\hat{g}(x,w) \gtreqless w \quad as \quad p(x) \gtreqless w. \tag{4.4}$$

PROOF: The monotonicity properties of $\hat{g}$ follow from those of g via (4.3). For (4.4), suppose $p(x) > w$. Then by (4.1) there is some $y < x$ such that $p(y) = w$. Hence, using (4.3), $\hat{g}(x,w) = \hat{g}(x,p(y)) = g(x,y) > g(y,y) = p(y) = w$. The cases for $p(x) < w$ and $p(x) = w$ are similar. *Q.E.D.*

Given a realization x of i's private signal X_i, the relation between the price W_i and i's conditional valuation of the objects is described by the function $\hat{g}(x,\cdot)$ as illustrated in Figure 1. The monotonicity of $\hat{g}(x,\cdot)$ is displayed in the figure and the relationship (4.4) is represented graphically by the relative heights of the function and the 45° line. For pictorial convenience, the range of W_i has been represented as a convex set and the function has been shown to be continuous and strictly increasing. These extra properties are not used in the arguments given below.

Suppose, hypothetically, that bidder i is given the opportunity to observe his price W_i before submitting his bid. Suppose that $X_i = x$ and that the price is $W_i = w < p(x)$. Then it is apparent from Figure 1 that i will be well pleased with his bid $p(x)$, because the price w is less than his estimated valuation of the objects $\hat{g}(x,w)$, and his bid causes him to win an object at that price. If the price were $W_i = w' > p(x)$, bidder i would still be pleased with his bid. In this case, the

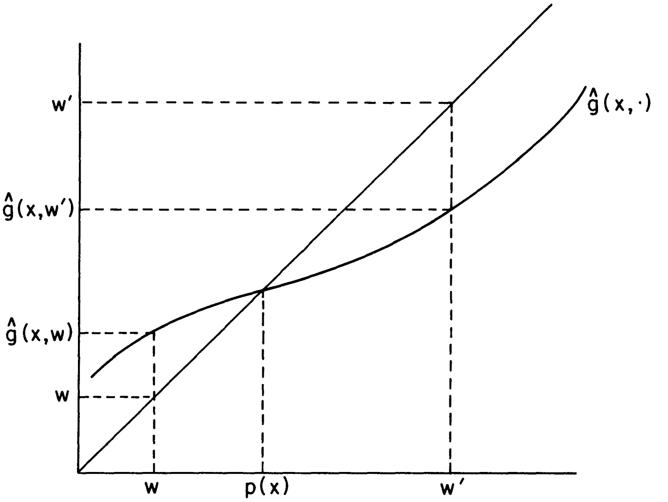

FIGURE 1

price w' exceeds his estimated valuation $g(x, w')$, and the bid $p(x)$ results in no purchase at this price.[7] The same global analysis applies to each possible realization x of X_i; so the following result has been proved.[8]

[7]In case of ties (i.e., $W_i = p(x)$), the analysis is much more subtle. Suppose, for example, that $k = 1$, that $n \geqslant 3$, and that the number of bidders with whom i has tied is m ($m \geqslant 1$). Large values of m indicate that more competitors have received favorable news than do small values of m. Indeed, conditional on $W_i = p(x)$, the random variable m has the MLRP! Notice, however, that i's chance of winning the tiebreaker is $1/(1 + m)$, so his chance of winning is highest when the news is least favorable! (Kreps [17] has pointed out a similar problem in REE models when traders, who compute excess demand correspondences, fail to account for the information conveyed by the auctioneers' choice of a point in their demand sets.) My assumptions sidestep this problem by ensuring that for all x: $P\{m > 1 \mid W_i = p(X_i) = x\} = 0$.

[8]Theorem 4.3 and, indeed, all the qualitative results of Sections 3–6 can easily be generalized to cover discrete, continuous, and mixed distributions $\{F(\cdot \mid z)\}$ with the (not necessarily strict) MLRP by means of a simple device. Let $\hat{X}_i = (X_i, U_i)$ where $U_1, \ldots, U_n$ are randomizing variables, independent of everything, and uniformly distributed on [0, 1]. Define the lexicographic order on $\mathbb{R}^2$ by $(x, u) \hat{>} (x', u')$ if either $x > x'$ or $x = x'$ and $u > u'$. Let $\hat{Y}_i$ be the kth order statistic in $\{\hat{X}_j \mid j \neq i\}$ using the lexicographic order. Let y be an atom of Y_i. A remarkable fact is this: if $u > u'$ then $\{\hat{Y}_i = (y, u)\}$ is strictly more favorable than $\{\hat{Y}_i = (y, u')\}$, despite the fact that the U_j's convey no information about Z!

To see why this is so, define three random variables U_{-i}, m_1, and m_2 by: $\hat{Y}_i = (Y_i, U_{-i})$, $m_1 = \#\{\hat{X}_j \mid X_j = Y_i, j \neq i, U_j < U_{-i}\}$, and $m_2 = \#\{\hat{X}_j \mid X_j = Y_i, j \neq i, U_j > U_{-i}\}$. For any fixed realization of X_i and Y_i, larger values of m_1 and smaller values of m_2 reflect relatively good news, because they indicate that many losers (m_1) have observed the best possible news that losers can observe and that few winners (m_2) have observed the worst possible news for winners. It is straightforward to check that U_{-i} has the MLRP as a signal about m_1 and as a signal about $-m_2$; so, for fixed Y_i, large values of U_{-i} represent relatively good news about Z.

Replacing each of the X_i's, Y_i's, x's and y's in Sections 3–6 by $\hat{X}_i$, $\hat{Y}_i$, $\hat{x}$, and $\hat{y}$, respectively, all the results go through (except that some strict inequalities become weak ones). The effect of the U_j's is to cause bidders to randomize in exactly those cases where ties would otherwise be a problem, and to account for and eliminate the "winner's curse" effect described in note 7.

This device of using an auxiliary randomizing variable and the lexicographic order to give a unified treatment of discrete, continuous, and mixed distributions appears to be widely applicable in bidding theory and related areas. For another application, see Engelbrecht-Wiggans, Milgrom, and Weber [6].

THEOREM 4.3: *Suppose that each of i's competitors adopts the strategy p defined by* (3.3). *Then for every possible realization x of* X_i *and w of* W_i

$$p(x) \in \arg\max_b E\left[u(Z, X_i, W_i)1_{\{W_i < b\}} \mid X_i = x, W_i = w\right].$$

Theorem 3.1 can now be proven as a simple corollary of Theorem 4.3.

PROOF OF THEOREM 3.1: By Theorem 4.3, p is the optimal response to the opposing strategies among all functions of X_i and W_i. Hence, it is optimal among all functions of X_i alone. *Q.E.D.*

Nothing has yet been said which guarantees that the slope of $\hat{g}(x, \cdot)$ is less than one. It is quite possible that i's valuation of the object does rise as fast or faster than the prices in some regions of the graph, as depicted in Figure 1. Theorem 4.3, however, guarantees that i's underlying "demand curve" is downward sloping, that is, he would demand one object at any price below $p(x)$ and no objects at higher prices.

Let us specialize, for the moment, to the case $k = 1$. Consider the problem faced by an $n + 1$st trader whose information is garbled compared to that of trader 1. Formally, this means that the conditional joint distribution of $(Z, X_2, \ldots, X_n)$ given X_1 and X_{n+1} does not depend on the value of X_{n+1}. Let $v(Z, X_{n+1}, b)$ denote $n + 1$'s payoff when he acquires an object for price b. As with the other bidders, $n + 1$'s payoff is zero when he acquires nothing. I shall assume that $n + 1$ has no transactions motive for trading with bidder 1, that is,

$$P\{u(Z, X_1, \cdot) \geqslant v(Z, X_{n+1}, \cdot)\} \equiv 1. \tag{4.5}$$

For example, in the special case $u(Z, X_1, b) \equiv v(Z, X_{n+1}, b) \equiv Z - b$, condition (4.5) is satisfied: bidder 1 assigns as great a dollar value to the object as does $n + 1$ for every possible quality level Z, and he is not more risk averse than $n + 1$.

Suppose the n bidders adopt the strategy p while bidder $n + 1$ observes X_{n+1} and then tenders a bid of b. This bid wins in two important cases: the case $p(X_1) < W_1 < b$ and the case $W_1 < p(X_1) \leqslant b$. (The events $\{W_1 = b\}$ and $\{W_1 = p(X_1)\}$ are null so the corresponding cases can be ignored.) Figure 2 shows that in each case, bidder $n + 1$ will be unhappy with his bid.

Suppose, hypothetically, that bidder $n + 1$ is informed of X_1 and W_1 in addition to X_{n+1}. Then by (4.5) and the garbling assumption, $n + 1$'s reservation price will be bounded above by $\hat{g}(X_1, W_1)$. The case $W_1 < p(X_1) \leqslant b$ is represented on the figure by $W_1 = w$ and $X_1 = x$. In this case, the price facing bidder $n + 1$ is $p(x_1)$, which is larger than $\hat{g}(x, w)$, which in turn bounds $n + 1$'s reservation price. Consequently, bidder $n + 1$ will regret his winning bid of b in this case. Similarly, the case $p(X_1) < W_1 < b$ is represented on the figure by $W_1 = w'$ and $X_1 = x$. Once again, the price faced by $n + 1$ (in this case, w') exceeds the bound $\hat{g}(x, w')$ on his reservation price. So $n + 1$ always regrets making a winning bid when he is told X_1 and W_1.

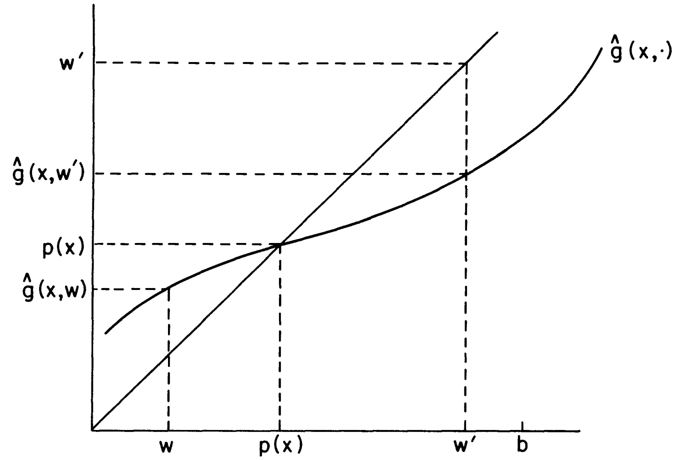

FIGURE 2

The foregoing argument can be generalized to the case of k objects under the following conditions.

(4.6) For $1 \leqslant i \leqslant k$, the conditional joint distribution of $Z, X_1, \ldots, X_n$ given X_i and X_{n+1} does not depend on the value of X_{n+1}.

(4.7) For $1 \leqslant i \leqslant k$ and for all b, $P\{u(Z,X_i,b) \geqslant v(Z,X_{n+1},b)\} = 1$.

Condition (4.6) is the "garbling" condition and (4.7) is the "no transactions motive" condition.

Let i^* denote the least optimistic individual among bidders 1 through k: $X_{i^*} = \min(X_1, \ldots, X_k)$. Bidder $n+1$ can win one of the k objects only by bidding more than the better informed bidder i^*, and hence paying more than i^* would be willing to pay. I show in the Appendix that if $n+1$ were told the values of X_{n+1}, X_{i^*}, i^* and W_{i^*}, he could still do no better than to bid zero and guarantee a payoff of zero. The conclusions reached above can be summarized as follows.

THEOREM 4.4: *Let W denote the kth highest bid among bidders* $1, \ldots, n$ *and assume that* (4.6) *and* (4.7) *hold. Then for all possible values x of X_{n+1},*

$$\sup_b E\left[v(Z,X_{n+1},W)1_{\{W<b\}} \,|\, X_{n+1} = x\right] = 0.$$

The Vickrey auction studied here has the same sort of limiting properties for large n as was demonstrated for the "first-price" auction by Wilson and Milgrom. Let W^n be the $k+1$st highest bid in the auction with n bidders. Define the *value* V^n of the objects by

(4.8) $$V^n(Z,X) = \sup_{i \leqslant n}\left(\sup\{b \,|\, u(Z,X_i,b) \geqslant 0\}\right).$$

The inner supremum represents bidder i's reservation price on the assumption that he has perfect information about Z. Thus, V^n is the most that any bidder would be willing to pay under perfect information.

THEOREM 4.5: *Let the number of objects k be fixed. Then $|V^n - W^n|$ converges to zero in probability as $n \to \infty$ if and only if for each two possible values z' and z of Z with $z' > z$,*

$$\inf_x \frac{f(x \mid z)}{f(x \mid z')} = 0. \tag{4.9}$$

Theorem 4.5 can be proved by a variation of the argument given in Milgrom [**19**]. The theorem indicates that prices may give a good approximation of value when n is large.

5. A REVEALING EXAMPLE

The price formed by the Vickrey auction is not, in general, a fully revealing price. To see this, one need only recognize that the price is simply an order statistic from a certain set of random variables, and order statistics can be sufficient only under very restrictive distributional assumptions. Nevertheless, to emphasize that the Grossman-Stiglitz information acquisition paradox arises from their price-taking behavior assumption rather than from the fully revealing prices, it is useful to study a special example of the Vickrey auction model for which the price is "fully revealing."

Suppose that there is only one object being sold, that is, $k = 1$. Let $u(Z, X_i, b) = Z - b$, so that the object being sold is worth Z to each and every bidder. However, no bidder knows the value of Z. Suppose that the conditional distribution of each X_i given Z is uniform on $[0, Z]$:

$$f(x \mid z) = \begin{cases} 1/z & \text{for } 0 \leqslant x < z, \\ 0 & \text{otherwise.} \end{cases} \tag{5.1}$$

Finally, suppose that the prior G for Z is uniform on $[0, M]$:

$$G(s) = \begin{cases} 0 & \text{for } s \leqslant 0, \\ s/M & \text{for } 0 \leqslant s \leqslant M, \\ 1 & \text{for } M \leqslant s. \end{cases} \tag{5.2}$$

Using (3.3) and (3.5), the symmetric equilibrium bidding strategy p^n for an n bidder auction can be computed:

$$p^n(x) = \begin{cases} \dfrac{\ln(M/x)}{x^{-1} - M^{-1}} & \text{if } n = 2, \\[2ex] \dfrac{(n-1)(x^{2-n} - M^{2-n})}{(n-2)(x^{1-n} - M^{1-n})} & \text{if } n > 2. \end{cases} \tag{5.3}$$

As always, the equilibrium strategy is an increasing function. However in this case, the variable Y_i is a *sufficient statistic* for the variables $\{X_j \mid j \neq i\}$, so the price variable $W_i = p^n(Y_i)$ is also a sufficient statistic. Thus, the price W_i reveals all of the information held by i's competitors. Unlike the fully revealing prices in previous rational expectations models, *W_i does not also reveal i's private information.*

Using (5.1)–(5.3), one can compute the expected profit $\pi(n)$ of an individual bidder in an n bidder auction ($n \geqslant 2$).

Now instead of assuming that each bidder freely observes his own private information before taking any discretionary action, let us suppose that the game proceeds in two stages. At the first stage, each bidder must choose whether or not to observe his private signal. There is a cost c associated with making the observation. Then each bidder is told the number of bidders m who have made an observation. At the second stage, each informed bidder i makes a bid which may depend on m and X_i and each uninformed bidder makes a bid which may depend only on m.

A strategy for each bidder consists of a first stage decision and a second stage decision rule. If the bidder chooses to remain uninformed, the second stage rule specifies a bid as a function of m. If the bidder chooses to become informed, the rule specifies a bid as a function of both m and the bidder's private signal.

Let us begin the analysis of this game by studying the case: $\pi(n) > c$. Then there is a symmetric equilibrium in which each bidder chooses to become informed and adopts the second stage decision rule described in Table I.

To verify that the specified strategies form an equilibrium consider first the problem faced by a bidder at the second stage, after the number of informed bidders m has been determined. The optimality of the zero bid by uninformed bidders when $m \geqslant 2$ is guaranteed by Theorem 4.4. It is straightforward to check that the remaining specifications are best responses for all m, so these strategies leave the second stage sub-game in equilibrium. Notice that when $m = 1$ the informed bidder's expected payoff is $E[Z]$. Let us therefore define $\pi(1) = E[Z]$.

At the first stage, a bidder who remains uninformed can anticipate a payoff of zero. If he becomes informed, his expected payoff is $\pi(n) - c > 0$. Therefore, all bidders will choose to become informed. The following has been proven.

THEOREM 5.1: *If $\pi(n) > c$, then there is a symmetric equilibrium in which each bidder chooses to become informed and adopts the second stage decision rule given in Table I. (The same strategies are in equilibrium when $\pi(n) = c$.)*

TABLE I

SECOND STAGE EQUILIBRIUM BIDS

Is i informed?	Signal	Number Informed	Bid
No.	N/A	0	$E[Z]$
No.	N/A	$m \geqslant 1$	0
Yes.	x	1	$E[Z \mid X_i = x]$
Yes.	x	$m \geqslant 2$	$p^m(x)$

When $\pi(n) < c < \pi(1)$, two sorts of equilibria suggest themselves. In the first, let n^* be the largest integer not exceeding n for which $\pi(n^*) \geqslant c$. Then there is an asymmetric equilibrium in which bidders 1 through n^* choose to become informed while the other bidders remain uninformed. For the second stage of this equilibrium, all bidders adopt the rule given in Table I.

There is also a symmetric equilibrium in which each bidder randomly decides whether or not to become informed. To see this, let us define $f: [0,1] \to \mathbb{R}$ as follows.

$$f(q) = \sum_{r=0}^{n-1} \binom{n-1}{r} q^r (1-q)^{n-1-r} \pi(r+1). \tag{5.4}$$

The number $f(q)$ is the expected second stage payoff to an informed bidder when each of his competitors chooses to become informed with probability q. Since f is continuous and since $f(0) = \pi(1) > c > \pi(n) = f(1)$, there is some q^* $(0 < q^* < 1)$ for which $f(q^*) = c$. When each bidder chooses to become informed with probability q^*, the expected payoff from both stages combined is zero to every bidder.

THEOREM 5.2: *If $\pi(1) > c > \pi(n)$, then there is a symmetric equilibrium in which each bidder becomes informed at the first stage with probability q^* and adopts the second stage decision rule given in Table I.*

The proof of Theorem 5.2 is straightforward. For completeness, note that when $c \geqslant \pi(1)$, there is a symmetric equilibrium in which no bidder becomes informed and each adopts the second stage strategy given in Table I. Thus, unlike the Grossman-Stiglitz model, there is no level of c which is inconsistent with equilibrium, despite the fully revealing prices.

6. VARIATIONS OF THE BASIC MODEL

This section addresses three questions which arise naturally in studying the basic model. (i) In real auctions, the seller commonly sets a minimum price. Can a minimum price be easily introduced into this model? (ii) Can the assumption that bidders observe conditionally independent signals be weakened without destroying the qualitative results? (iii) Does the model have other equilibria, in addition to the symmetric equilibrium?

It is, in fact, fairly easy to introduce a seller reservation price into the model, and the process of doing so gives additional insights into the nature of the equilibrium. Let r be the minimum price set by the seller ($r \geqslant 0$). If the $k+1$th highest bid is less than r, then all bids exceeding r are declared to be winning bids, and each winner acquires one object at the price r. Obviously, if $p(x) \geqslant r$ for all possible x, then the minimum price has no effect on the equilibrium.

THEOREM 6.1: *Suppose there is some x such that $p(x) < r$. Then the bidding* *game has a symmetric equilibrium strategy* p^* *given by*

$$p^*(x) = \begin{cases} p(x) & \text{if } x \geqslant x^*, \\ 0 & \text{otherwise,} \end{cases} \tag{6.1}$$

where x^* *is defined by*

$$x^* = \sup\{x \mid E[u(Z, X_i, r) \mid X_i = x, Y_i < x] < 0\}. \tag{6.2}$$

Just as Theorem 3.1 was proved as a corollary of a "no regret" theorem (Theorem 4.3), the foregoing result is also the corollary to a "no regret" theorem. Define i's price variable W_i^* by

$$W_i^* = \max(r, p^*(Y_i)). \tag{6.3}$$

THEOREM 6.2: *Suppose that i's competitors adopt the strategy p^*. Then for every x in the range of X_i and every w in the range of W_i^**

$$p^*(x) \in \arg\max_b E\left[u(Z, X_i, W_i^*) 1_{\{W_i < b\}} \mid X_i = x, W_i^* = w\right]. \tag{6.4}$$

PROOF: Replacing W_i by W_i^* in expression (4.2), one can define a function $\bar{g}(x, w)$ which is analogous to $\hat{g}(x, w)$. Figure 3 then illustrates i's optimization problem for the two possible cases: when he observes a signal $x < x^*$ and when he observes a signal $x' > x^*$. (The case $X_i = x^*$ is both trivial and null.) In each case, it is apparent from the kinds of reasoning used before that $p^*(x)$ and $p^*(x')$ are indeed optimal even if the value of W_i^* is known. *Q.E.D.*

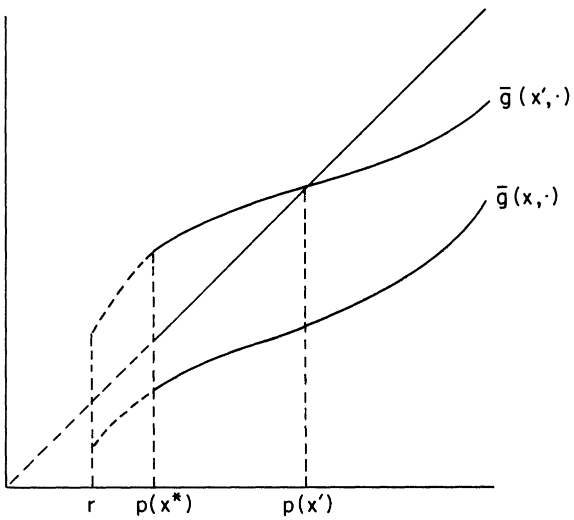

FIGURE 3

The zero profit result (Theorem 4.4) can also be proved for this more general model. The proof follows the lines developed in Section 4.

Let us turn next to the issue of dependent information systems. The independence Assumption A2 was used directly to prove Theorems 2.4 and 3.2. Both of these theorems can be proved under a variety of assumptions, of which the following is perhaps the simplest.[9]

ASSUMPTION A2*—*Exchangeability*: *Conditional on Z, the random variable Z^* has the MLRP. Conditional on Z^*, the variables $Z, X_1, \dots, X_n$ are independent. Also, conditional on Z^*, the variables $X_1, \dots, X_n$ are identically distributed and have the MLRP.*

As an example of variables satisfying Assumption A2*, let $e_0, \dots, e_n$ be independent normally distributed random variables. Let $e_1, \dots, e_n$ be identically distributed, and assume that the e_i's are independent of Z, so that they represent "pure noise." Finally, let

$$X_i = Z + e_0 + e_i. \tag{6.5}$$

Then, conditional on Z, the X_i's are dependent due to the common error term e_0. However, taking $Z^* = Z + e_0$, these X_i's satisfy Assumption A2*.

There are two ways to show that Assumption A2* is sufficient for this model. One way is to trace the proofs through from the beginning. Mathematically, however, a simpler way is to define a new game which is strategically equivalent to the old game under Assumption A2*. The new game will satisfy all of the Assumptions A1–A6, so all the theorems will continue to hold. The trick runs as follows.

Let Z^* play the role of Z in the new game. Conditional on Z^*, the signals are independent, so they satisfy Assumption A2. Replace the utility function u of the old game by

$$\hat{u}(z,x,b) = E\big[u(Z,x,b) \,|\, Z^* = z\big]. \tag{6.6}$$

It is then routine to check that Assumptions A3–A6 are satisfied, so the entire development goes through.

[9] If the conditional joint density function $f(x_1, \dots, x_n, z)$ is twice continuously differentiable and symmetric in $(x_1, \dots, x_n)$, then another condition which includes Assumption A2* and which leads to the same theorems is

$$\frac{\partial^2 \ln f}{\partial z\, \partial x_i} \geqslant 0,$$

$$\frac{\partial^2 \ln f}{\partial x_i\, \partial x_j} \geqslant 0 \quad \text{for} \quad i \neq j.$$

The first differential condition is equivalent to the MLRP assumption. The second directly generalizes the conditional independence assumption. For details, see Milgrom **[19]** and Milgrom and Weber **[21]**.

The third and final question to be addressed in this section is the question of other equilibria. Unfortunately, Vickrey auctions generally have a plethora of equilibria, many of which are quite pathological. For example, suppose that $n = 2$, $k = 1$, $u(Z, X_i, b) = X_i - b$, and X_i is distributed on $[0, 1]$. Then the equilibrium of Theorem 3.1 is $p(x) = x$, and this is indeed Vickrey's dominant strategy equilibrium. Another equilibrium arises when bidder 1 always bids 3, regardless of x, and bidder 2 always bids 0. The number of implausible equilibria of this sort is quite large. None of these equilibria, however, is *perfect*, as defined by Selten [**31**], that is, these equilibria crumble if each player assigns some small probability to the event that his competitor may make a mistake. For example, in the equilibrium cited above, bidder 1 must fear that 2 will use his dominant strategy $p(x) = x$, while 2 has left himself no hope of earning any positive payoff, even if 1 errs.

Now suppose that the utility functions are $u(Z, X_i, b) = Z - b$, so that there may be no dominant strategy equilibrium. Let $n = 2$ and $k = 1$, let Z have some nondegenerate prior distribution, and let the X_i's be normally distributed with mean Z and variance one.

THEOREM 6.3: *Let $h : \mathbb{R} \to \mathbb{R}$ be any increasing, surjective function. Consider the strategies given by*

$$p_1(x) = E\left[Z \mid X_1 = x, X_2 = h(x)\right] \quad \text{and} \tag{6.7a}$$

$$p_2(x) = E\left[Z \mid X_1 = h^{-1}(x), X_2 = x\right]. \tag{6.7b}$$

Then for every possible value x of X_i and w of W_i,

$$p_i(x) \in \arg\max_b E\left[(Z - W_i) 1_{\{W_i < b\}} \mid X_i = x, W_i = w\right]. \tag{6.8}$$

In particular, (p_1, p_2) is an equilibrium pair.

PROOF: Simply observe that $p_1(x) = p_2(h(x))$ and apply the standard graphical argument. *Q.E.D.*

The equilibria of Theorem 6.3 are all perfect equilibria, yet some resemble rather closely the pathological equilibria discussed above. For example, let $h(x) = x + a$. Then as a grows large, 1's bid will converge to the upper bound of the distribution of Z while 2's bid will converge to the lower bound, irrespective of their signals. Still, I can find no completely convincing way to rule out these strange equilibria.

7. CONCLUSION

This paper has developed a bidding model with explicit rational expectations features. The model has several desirable properties which are missing in many rational expectations models. These are: (i) The bidders act as price takers because they cannot, in fact, influence their prices. I argued in the introduction that REE traders can often dramatically influence the prices they face. (ii) The prices vary directly with underlying qualities. Higher prices indicate better quality. (iii) There is no tension between the informational efficiency of prices, the incentive to gather information, and the possibility of reaching an equilibrium. (iv) The process by which prices are formed is made explicit. (v) For most model specifications, the price at which trading takes place is *not* fully revealing. (vi) All profits earned by the bidders arise either as gains from trade or as a result of speculation based on good private information. Poorly informed speculators can only lose.

The bidding model also has two principal shortcomings relative to REE models. First, it is a one-sided market model; the sellers play a purely passive role. Second, the buyers have very limited options: each can acquire only one of the objects being sold. Each of these weaknesses needs to be addressed by further research.

The Vickrey auction model provides a convenient device for relating bidding theory to REE theory, because the model shares many features (such as price taking behavior) with the latter theory. However, qualitatively similar features do emerge from other kinds of bidding models. In models of the discriminatory auction, for example, where each winner pays a price equal to his own bid, these prices may aggregate information. Still, there is no tension between information gathering and informationally efficient prices. For these models, even though the bidder's private information is reflected in the price he pays, the tension is broken by the absence of any price taking behavior assumption.

Various other bidding models merit study. The oral auction, for example, is one which has not been studied when the objects being traded are of uncertain quality. When the quality of the objects is known, Vickrey argued that the oral auction is strategically equivalent to the highest rejected bid sealed-tender auction. However, this equivalence breaks down when quality is uncertain, because bidders in the oral auction may be able to learn about quality from the bidding behavior of their competitors.

In studying these and other processes of price formation, it is important that researchers admit the possibility that the outcome of trading may depend critically on the nature of the trading process, and that the variety of possible outcomes may not be representable by any single model.

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Manuscript received January, 1980; revision received September, 1980.

APPENDIX: PROOF OF THEOREM 4.4

LEMMA 1: *The conditional distribution of Z given $(i^*, X_i^*, W_i^*, X_{n+1})$ does not depend on the value of X_{n+1}.*

PROOF: Since i^* and W_i^* are deterministic functions of $X_1, \dots, X_n$, it follows from the garbling condition that the joint distribution of (Z, i^*, W_i^*) given (X_i, X_{n+1}) does not depend on X_{n+1} (for $1 \leqslant i \leqslant k$).

Hence, the conditional distribution of Z given $(i^*, X_i, W_i^*, X_{n+1})$ does not depend on X_{n+1} (by Bayes' Theorem). In particular, in the event $\{i^* = i\}$, the conditional distribution of Z given $(i^* = i,$

X_i^*, W_i^*, X_{n+1}) does not depend on X_{n+1}. But the events $\{i^* = i\}$ as i varies are exhaustive and mutually exclusive. *Q.E.D.*

Now let us define a function g^* by

$$g^*(x, y) = \sup\{b \mid E[u(Z, x, b) \mid X_i^* = x, Y_i^* = y] \geqslant 0\}. \tag{A.1}$$

Also, define (analogously to (4.2))

$$\hat{g}^*(x, w) = \sup\{b \mid E[u(Z, x, b) \mid X_i^* = x, W_i^* = w] \geqslant 0\}. \tag{A.2}$$

Then the functions are related by

$$g^*(x, y) = \hat{g}^*(x, p(y)). \tag{A.3}$$

The conditional joint density of (X_i^*, Y_i^*) given Z evaluated at (x, y, z) is

$$f(x, y \mid z) = \begin{cases} k(n-k)f(x \mid z)f(y \mid z)(1 - F(x \mid z))^{k-1}F^{n-k-1}(y \mid z) & \text{for } y \leqslant x, \\ k(n-k)f(x \mid z)f(y \mid z)(1 - F(y \mid z))^{k-1}F^{n-k-1}(y \mid z) & \text{for } y \geqslant x. \end{cases} \tag{A.4}$$

Holding x fixed, this density (viewed as a function of y given z) has the MLRP. Similarly, holding y fixed, the density of x has the MLRP. Thus, by an extended version of Theorem 2.1, posterior beliefs are monotonically increasing in X_{i^*} and Y_{i^*}. This leads to the following result.

LEMMA 2: *The function $g^*(x, y)$ is increasing in x and nondecreasing in y.*

The joint density of X_i and Y_i (note that the stars have been dropped) is

$$f(x, y \mid z) = \frac{(n-1)!}{(k-1)!\,(n-k-1)!} f(x \mid z)f(y \mid z)(1 - F(x \mid z))^{k-1}F^{n-k-1}(y \mid z). \tag{A.5}$$

Observe that $f^*(x, x \mid z) = f(x, x \mid z)k!(n-k)!/(n-1)!$ It follows the conditional distribution of Z given $X_{i^*} = x$ and $Y_{i^*} = x$ is identical to the conditional distribution given $X_i = x$ and $Y_i = x$. (To prove this, simply apply Bayes Theorem.) This leads to the next lemma.

LEMMA 3: $g^*(x, x) \equiv g(x, x)$.

In view of Lemmas 2 and 3, the proof of Theorem 4.2 can be mimicked to establish the following result.

LEMMA 4: *The function $\hat{g}^*(x, w)$ is increasing in x and nondecreasing in w. Moreover, the following relationships hold*:

$$\hat{g}^*(x, w) \gtreqless w \quad \text{as} \quad p(x) \gtreqless w.$$

With these four lemmas in hand, Theorem 4.4 is proved as follows. Let b be any arbitrary nonnegative number. The computation of $n + 1$'s expected payoff when he bids b goes as follows.

$$\begin{aligned} & E[v(Z, X_{n+1}, W)1_{\{w \dot{<} b\}} \mid X_{n+1}] \\ & \overset{a}{\leqslant} E[u(Z, X_{i^*}, W)1_{\{W \dot{<} b\}} \mid X_{n+1}] \\ & \overset{b}{\leqslant} E\big[E[u(Z, X_{i^*}, W) \mid X_{n+1}, i^*, X_{i^*}, W_{i^*}]1_{\{W \dot{<} b\}} \mid X_{n+1}\big] \\ & \overset{c}{\leqslant} E\big[E[u(Z, X_{i^*}, W) \mid i^*, X_{i^*}, W_{i^*}]1_{\{W \dot{<} b\}} \mid X_{n+1}\big] \\ & \overset{d}{\leqslant} E\big[E[u(Z, X_{i^*}, W) \mid X_{i^*}, W_{i^*}]1_{\{W \dot{<} b\}} \mid X_{n+1}\big] \\ & \overset{e}{\leqslant} E\big[E[u(Z, X_{i^*}, W_{i^*}) \mid X_{i^*}, W_{i^*}]1_{\{p(X_{i^*}) < W_{i^*} \dot{<} b\}} \mid X_{n+1}\big] \\ & \quad + E\big[E[u(Z, X_{i^*}, p(X_{i^*})) \mid X_{i^*}, W_{i^*}]1_{\{W_{i^*} < p(X_{i^*}) \dot{<} b\}} \mid X_{n+1}\big] \\ & \overset{f}{\leqslant} 0. \end{aligned} \tag{A.6}$$

Step a follows from (4.7), the "no transactions motive" condition. Step b is justified by the properties of conditional expectations. Steps c and d apply the garbling hypothesis (4.6) and the symmetry assumption A1, respectively. Step e splits consideration into two cases: the case $W_{i^*} > p(X_{i^*})$ (for which $W = W_{i^*}$) and the case $W_{i^*} < p(X_{i^*})$ (for which $W = p(X_{i^*})$). The third case ($W_{i^*} = p(X_{i^*})$) is null and so can be safely omitted (see note 4). By Lemma 4 and the definition (A.2) of g^*, the integrand in e is everywhere nonpositive. This fact justifies step f.

Hence, it is proved that no bid for player $n + 1$ earns a positive expected payoff. Since setting $b = 0$ earns a nonnegative expected payoff (by (A4)), Theorem 4.4 is proved.

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