

THE VALUE OF INFORMATION IN A SEALED-BID AUCTION*

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In this paper, we explore bidders' incentives to gather information in auctions, when there is one bidder with only public information and another with some private information. We find that the bidder with only public information makes no profit at equilibrium, while the bidder with private information generally makes positive profits. Moreover, the informed bidder's profits rise when he gathers extra information, and the increase is greater when the information is collected overtly than when it is collected covertly. When the uninformed bidder can observe some of the better-informed bidder's information, he prefers to make his observations covertly. If the seller has access to some of the better-informed bidder's information, or if he has affiliated information of his own, he can raise the expected price by adopting a policy of making that information public. However, there are cases where a policy of publicizing his information would lower the expected price. The distinguishing feature of these latter cases seems to be that the seller's information is complementary to the information of the better-informed bidder.

1. Introduction

Periodically, the United States Department of the Interior sells drilling rights for oil and gas on federally-owned properties by means of a sealed-bid auction. The sums involved in these sales are often large. For example, the total amount paid for the rights to 147 tracts in the Gulf of Mexico in September, 1980 was \$2.8 billion. Two individual tracts drew winning bids in excess of \$160 million.

To a first approximation, the value of the rights on a tract — as determined by the amount of oil in the ground, its depth and pressure, future prices for refined petroleum products, etc. — is the same to each bidder. However, the bidders may differ considerably in their information about this value. Some may have access only to publicly available geological data concerning the site. Others may have additional proprietary information, perhaps resulting from work on a nearby tract or from a privately commissioned survey. Our purpose in this paper is to study how the nature of the information available to a bidder affects his bidding strategy, his opponents' bidding strategies, the profits earned by the various bidders, and the revenue accruing to the seller.

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In statistical decision problems, information never has a negative value to the decision-maker. At worst, irrelevant information can be ignored. More generally in multi-person settings, information that is gathered *covertly* cannot cause a competitive response, and therefore cannot have a negative value. However, information that is gathered *overtly* can have a positive, negative, or zero value.¹

Akerlof's (1970) well-known 'lemons' model provides an example of information with negative value. In that model, the seller of a used car knows more about the quality of his car than does the buyer, and the buyer knows that the seller is so informed. Consequently the buyer worries that the car is being sold because it is a 'lemon'. If the seller did not have superior information and the buyer knew that, then the car could be sold for its expected value,² but as things stand, the car can be sold only at a discount. Other similar examples include that of Spence (1974), where workers who know more about their ability than their employers do can be drawn into harmful competition, and that of Milgrom and Roberts (1982), where a firm with superior cost and demand data is trapped into playing a disadvantageous limit-pricing game.

In these examples, when one agent is known to have extra information, the others alter their behavior in a way that is unfavorable to him. This kind of response does not reflect any underlying general principle; acquiring information can sometimes cause favorable changes in the behavior of others.

For example, in this paper we study a two-bidder auction³ in which bidder A has some proprietary information while bidder B has only public information concerning the value V of the object being sold. We show that A prefers to do any further information gathering *overtly*, rather than *covertly*, because B's response to A's extra information is to bid more timidly. Bidder B, however, prefers to gather any additional information *covertly*.

The intuition behind these results is as follows. Since A is better informed than B, B must worry that his bid will win only if A holds a low estimate of V (that is, B is subject to the so-called 'Winner's Curse'). If A acquires additional information and B knows this, B must bid still more cautiously to avoid losing money. This reduction in B's bids benefits A. Of course, anticipation of B's response will cause A to adjust his strategy as well. However, the effects just described persist in the new equilibrium.

An entirely different line of reasoning applies when the more poorly informed bidder B has a chance to observe some of A's information. We have shown elsewhere [Engelbrecht-Wiggans et al. (1981), Milgrom (1979)] that B has an

¹An exception arises in two-person zero-sum games, where even overtly gathered information cannot have a negative value. For three-person zero-sum games and two-person general-sum games, information can have positive, negative, or zero value to a player at equilibrium.

²Actually, this argument requires that it be *common knowledge* [Aumann (1976), Milgrom (1981)] that the seller does not have superior information.

³Throughout this paper, we restrict our consideration to the standard sealed-bid auction, in which each bidder submits a non-negative bid and the object is sold to the highest bidder for the amount of his bid.

expected profit of zero at equilibrium if he holds only public information. Consequently, B would not benefit from observing some of A's information overtly, but could benefit from observing it covertly.

If one thinks of the seller as a player, the auction is a three-person constant-sum game: the bidders' profits plus the seller's revenues must add up to the value V . Consequently, it may be in the seller's interest to neutralize A's informational advantage in order to reduce A's profits and raise the selling price. It is then natural to ask: How should the seller manage any information to which he may be privy? The answer depends on the nature of the seller's information. Under some circumstances, that information may be complementary to the information already possessed by A. In these cases, a policy of publicly announcing the information may serve to accentuate A's informational advantage, raising his expected profits and depressing the expected selling price. Often, however, the seller's information is not complementary to A's information. For example, the seller may have access to a royalty report filed by A on a nearby tract, or the seller's information may be statistically 'linked' in some manner with A's information. In both of these cases, the expected selling price will rise when the seller adopts a policy of making his information public.

2. A formal model

We shall study a model with two bidders, A and B. Bidder B has only public information about the (non-negative) value V of the rights being sold, but bidder A observes some private signal X . Both bidders are assumed to be risk-neutral, and V is assumed to have a finite expectation.⁴

Since B observes no information, his bidding strategy can be described by a distribution G over the non-negative numbers, representing his random choice of a bid. For A, a (pure) strategy is a function mapping the possible values of his signal X into non-negative bids. When B uses the atomless strategy G and A observes $X = x$, A's best bid solves

$$\max_b G(b)(E[V|X=x] - b).$$

Since this problem depends on X only through $E[V|X]$, we can summarize A's information by the statistic $H = E[V|X]$. Note that H is itself a random variable, since its value depends on the realization of the random variable X . In this paper, we assume that H has an atomless distribution F , though we have shown elsewhere [Engelbrecht-Wiggans et al. (1981)] that this assumption has no qualitative effect on the equilibrium.

⁴This model was introduced by Wilson (1967) and has subsequently been studied in Engelbrecht-Wiggans et al. (1981), Weverburgh (1979) and Wilson (1975).

An equilibrium of this game is a pair of strategies (b^*, G^*) for A and B such that $b^*(h)$ solves A's problem for every realization h of H and G^* maximizes B's expected payoff, which is

$$\int \{E[V | b^*(H) \leq s] - s\} dG^*(s).$$

The unique equilibrium of this game has been determined in Engelbrecht-Wiggans et al. (1981).

Theorem 1. The two-person auction described above has a unique equilibrium (b^*, G^*) , where

$$b^*(h) = E[H | H \leq h] = h - \frac{\int_0^h F(s) ds}{F(h)},$$

$$G^*(b) = F(b^{*-1}(b)) = \Pr \{b^*(H) \leq b\}.$$

At equilibrium, the distribution of bids is the same for both bidders.

When bidder A observes $H = h$ and makes his equilibrium bid $b^*(h)$, he wins with probability $G^*(b^*(h))$. By Theorem 1, this probability is equal to $F(h)$. When he wins, he receives rights of expected value h , but pays only $b^*(h)$. So his conditional expected winnings when he observes $H = h$ are $F(h) \cdot (h - b^*(h)) = \int_0^h F(t) dt$. Integrating this over h and interchanging the order of integration leads to the following result:

Theorem 2. Bidder A's expected profit at equilibrium is given by

$$\int_0^{\infty} (1 - F(t))F(t) dt,$$

where F is the distribution of the statistic $H = E[V | X]$.

Let b be any bid made by bidder B in equilibrium. Since the bid distributions of A and B have identical supports, there must be some h such that $b = b^*(h)$. Hence, B's expected profit from the bid b is

$$F(h) \cdot E[V - b | H \leq h] = F(h) \cdot \{E[V | H \leq h] - b^*(h)\} = 0.$$

As this must hold for all of B's possible bids, B's expected profit at equilibrium is zero. This result is a special case of the following [Milgrom (1979)]:

Theorem 3. In an auction for an object of unknown value V , suppose there are two risk-neutral bidders A and B such that A's information partition is finer than B's. At any equilibrium of the auction game, B's expected payoff is zero.

Now suppose A has the opportunity to conduct a private survey of the site offered for sale. As observed previously, a covert survey cannot have negative value. We now show that an overtly conducted survey is actually preferred by A to a covertly conducted one. In fact, we prove that regardless of the outcome of the survey, bidder A would prefer that B know that a survey has been conducted.

Theorem 4. Let bidder A have the following three choices:

- (i) *Observe X only, and have B know that only X is observed.*
- (ii) *Observe X and Y, but have B act as if only X were observed.*
- (iii) *Observe X and Y, and have B know that both are observed.*

On the basis of expected profits A, (ii) is preferred to (i) and (iii) is preferred to (ii). Indeed, (iii) is preferred to (ii) for every possible realization of X and Y.

Proof. The preference for (ii) over (i) follows from A's ability to ignore Y if it is not helpful. To compare (iii) and (ii), let us consider the maximum payoff that can be obtained in each case. Let $H_1 = E[V|X, Y]$ and let F_1 be the distribution of H_1 .

Suppose that the particular realization of H_1 is h . Then A's problem in case (ii) is to maximize $G^*(b)(h - b)$. If h is in the range of H , then this problem is identical to the one arising in case (i) when the realization of H is h . From the analysis of that problem, we know that the optimal bid is $b = E[H|H \leq h]$, and the expected payoff is $\int_0^h F(t) dt$. If h lies above the support of H , it is not difficult to show that the optimal bid is $E[H]$, and that the expected payoff expression is the same as given above. Similarly, if h lies below the support of H , an optimal bid is zero and the expected payoff expression given above is still correct.

A similar argument for case (iii) shows that when $H_1 = h$ is observed, A's expected payoff is $\int_0^h F_1(t) dt$. We now claim the following:

$$0 \leq \int_0^h (F_1(t) - F(t)) dt \text{ for all } h.$$

A short proof of this claim can be given using the result of Rothschild and Stiglitz (1970) that this inequality holds if H_1 'is riskier than' H , i.e., if for every convex function u , $E[u(H_1)] \geq E[u(H)]$. But since $H = E[H_1|X]$, we have the following by Jensen's inequality:

$$E[u(H)] = E[u(E[H_1|X])] \leq E[E[u(H_1)|X]] = E[u(H_1)]. \quad \text{Q.E.D.}$$

Theorem 5. Let A observe X and Y and let B have the following four choices:

- (i) *Observe nothing, and have A know this.*
- (ii) *Observe X, and have A know this.*

- (iii) Observe $E[V|X] + \varepsilon$, where ε is a random variable independent of (V, X, Y) , and have A know this.
- (iv) Observe X , and have A bid as if B observed nothing.

On the basis of expected profits for B, (i) and (ii) are the same, and both (iii) and (iv) are preferred to (i) and (ii).

Proof. This result follows easily from Theorem 3 and the observation that, in any equilibrium, bidders have non-negative expected profits. Q.E.D.

Two of the comparisons in Theorem 5 are especially interesting. First, the preference for (iii) over (ii) indicates that a bidder may prefer less information to more if the information must be gathered overtly.⁵ This is another example of information with negative value. Second, the preference for (iv) over (ii) is a preference for covert information over overt information, which is opposite from the comparable preference of the well-informed bidder A.

In the case of oil drilling rights, if bidder A's information comes from his experience on a nearby tract and if he has been filing reports on that experience as part of the determination of royalties, the seller may be able to make some of A's information public. To model this possibility, suppose A observes X and reports Z to the seller, or equivalently that A observes (X, Z) and reports Z . Let $H = E[V|X] = E[V|X, Z]$ and let $F(\cdot|Z)$ be the conditional distribution of H given Z .

With this set-up, it follows from Theorem 2 that A's conditional expected profit after Z has been revealed is

$$\int_0^{\infty} F(h|Z)(1 - F(h|Z)) dh.$$

For each h , $F(h|Z)$ is a random variable (because it depends on Z) and its expectation is $E[F(h|Z)] = F(h)$, the marginal probability that $H \leq h$.

Theorem 6. If the seller establishes a policy of making the report Z public, then A's expected profits will fall and the seller's expected revenue will rise.

Proof.

$$\begin{aligned} E \left[\int_0^{\infty} F(h|Z)(1 - F(h|Z)) dh \right] &= \int_0^{\infty} E[F(h|Z)(1 - F(h|Z))] dh \\ &\leq \int_0^{\infty} E[F(h|Z)](1 - E[F(h|Z)]) dh \\ &= \int_0^{\infty} F(h)(1 - F(h)) dh, \end{aligned}$$

⁵Wilson (1975) gives a simple example illustrating that (iii) can be strictly preferred to (i) and (ii).

where we have again used Jensen's inequality. This proves that A's profits fall. By Theorem 3, B's profits are zero in any case. Consequently, the seller's expected revenue rises. Q.E.D.

Theorem 6 asserts that reporting Z is better than reporting nothing. But might there be a still better policy, of withholding Z under some circumstances, or providing only a summary of Z , or reporting it subject to some garbling process? If the seller were to make public some report Z' which is noisier than Z , then by Theorem 6, the seller could further benefit after reporting Z' by reporting Z , too. Thus we have the following result:

Corollary. Among all policies for reporting Z , the policy which maximizes the seller's expected revenue is the policy of always reporting Z completely.

If the seller has private information of his own, does it pay him to report it in order to dilute A's informational advantage? The following example shows that such a policy can be unwise. Suppose $X = V + \varepsilon$, where ε is a noise term independent of V . Suppose the seller observes $Y = \varepsilon$, and announces it publicly. The result is that A knows V exactly, but B knows nothing useful. Then by Theorem 4, A's profits rise, and the seller's revenue falls.

The difficulty in this example is that the seller's information Y and the buyer's information X are highly *complementary*. In other situations, X and Y may represent independent estimates of V , or they may be estimates which are subject to some common source of error. In these other cases, X and Y are *informational substitutes*, and a policy of making Y public will raise the seller's expected revenue.

To formalize this idea, let us assume that X and Y are real-valued and that the functions $h(x) = E[V|X=x]$ and $k(x, y) = E[V|X=x, Y=y]$ are increasing in x and y and differentiable in x . It then follows from Theorem 2 and a change of variables that if Y is not revealed, A's expected profit is

$$\int_0^{\infty} F_X(x)(1 - F_X(x))h'(x) dx.$$

If Y is announced, A's expected profit is

$$\int_0^{\infty} \int_0^{\infty} F_X(x|y)(1 - F_X(x|y))k_x(x, y) dx f_Y(y) dy,$$

where $k_x = \partial k / \partial x$.

The difference between these two expressions can be expressed as the sum of two effects. The *publicity effect* is defined by

$$P = \int_0^{\infty} h'(x) \{ F_X(x)(1 - F_X(x)) - E[F_X(x|Y)(1 - F_X(x|Y))] \} dx,$$

and the *weighting effect* by

$$W = \int_0^{\infty} E[(h'(x) - k_x(x, Y))F_X(x|Y)(1 - F_X(x|Y))] dx.$$

When Y is revealed, B learns something about X , and in that sense A 's information becomes less private. The publicity effect then works to reduce A 's expected profit: by Jensen's inequality the integrand, and therefore the effect, is always non-negative.

The weighting effect reflects how the presence of Y influences the weight accorded to X in estimating V . We say that X and Y are informational *substitutes* in estimating V if the weighting effect is positive, and are *complements* if the effect is negative. One condition that clearly implies that X and Y are substitutes is that for all x and y , $k_x(x, y) \leq h'(x)$. This situation arises, for example, when X , Y , and V have a joint normal distribution with non-negative partial correlations. In this case, it is always to the seller's advantage to release the information he holds.

The assumption that $k_x(x, y) \leq h'(x)$ for all x and y excludes several important models. For example, the assumption fails to hold when $k(x, y) = a(x^\alpha y^\beta)$; this case arises when X and Y are independent lognormal estimates of V , or when X is an estimate of the volume of natural gas in some geological formation and Y is an estimate of its pressure.

The random variables in the vector $Z = (X, Y)$ are *affiliated* if the joint density function $f(z)$ satisfies the inequality $f(z \vee z')f(z \wedge z') \geq f(z)f(z')$ for all z and z' , where \vee and \wedge denote the operations of coordinate-wise maximization and minimization. If X and Y are affiliated, then X increases stochastically in Y , i.e., $F_X(x|y)$ is non-increasing in y for all x . If, furthermore, $E[V|X=x, Y=y]$ is non-decreasing in both x and y , then it follows that $E[V|X=x]$ is non-decreasing in x . A detailed development of affiliation and its properties is given in Milgrom and Weber (1982).

In Theorem 7 we present a pair of assumptions which together imply that X and Y are informational substitutes:

Theorem 7. If X and Y are affiliated, and if $E[V|X=x, Y=y]$ is non-decreasing in x and y , then the seller's expected revenue at equilibrium is higher when Y is released than when Y is withheld. Moreover, no policy of garbling or partially reporting Y leads to higher expected revenue than the policy of always reporting Y fully.

Proof. Although we do not assume that $h(x)$ is differentiable, the publicity effect can be rewritten using $dh(x)$ in place of $h'(x)$. As before, this effect is non-negative.

The weighting effect can also be written without derivatives. Integrating by

parts and writing the resulting integral as an expectation,

$$\begin{aligned}
 W &= E[(h(X) - k(X, Y))(2F_X(X|Y) - 1)] \\
 &= E[E[(h(X) - k(X, Y))(2F_X(X|Y) - 1)|X]] \\
 &\geq E[E[(h(X) - k(X, Y))|X]E[(2F_X(X|Y) - 1)|X]] \\
 &= 0.
 \end{aligned}$$

The inequality holds because both of the factors in the conditional expectation are non-increasing in Y ; the final equality follows from the equation $E[(h(X) - k(X, Y))|X] = 0$. Hence the weighting effect is non-negative.

Consequently, when Y is revealed, A's profit falls, B's profit remains zero, and the seller's expected revenue rises. This proves the first part of the theorem. It remains only to note that, conditional on any garbling of Y , the conditions of the theorem are still satisfied. Therefore, the second part of the theorem follows from the first part. Q.E.D.

Theorems 6 and 7 concern *policies* which a seller might adopt, and which the bidders are assumed to perceive correctly. For example, if the seller's policy were to report Y only when it conveys favorable information, then any failure of the seller to report information would lead the bidders to infer that Y was unfavorable. We note that it is in fact common, in accordance with these theorems, for sellers in auction-like situations to obtain expert appraisals of the items being offered, and to commit themselves to making these appraisals public. For instance, when bonds are to be issued the bond rating agency's contract typically specifies that the appraisal must become part of the public record.

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